
Theory of Mutual Interference in Tandem Compressor Cascade*

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Two adjacent rows of blades, which constitute a tandem compressor cascade, interfere aerodynamically with each other, so that the deflection angle and losses are different from those of a system having no interference.

In order to calculate the interference effect on cascade performance, a method must be found for calculating the change of flow field due to interference.

This paper presents a theoretical method for determining the two-dimensional incompressible potential flow through an arbitrary tandem compressor cascade based on a singularity method. This theory is especially suitable for the case in which the relative position between the two blade rows varies, while the blade profiles and settings remain constant.

Results of sample calculation are shown and compared with the corresponding experimental results.

1. Introduction

For the design of a compressor cascade, which realizes a given velocity triangle with a large deflection angle, we must solve the problem, at first, as to which is more preferable, a usual single row cascade [Fig. 1 (a)] or a tandem cascade [Fig. 1 (b)].

The author compared the losses of both cascade systems to answer this problem, and found out that a tandem compressor cascade had less losses within the loading region of $C_L \sigma > 1.2^{(1)}$.

Two adjacent rows of blades, which constitute a tandem compressor cascade, interfere aerodynamically with each other, so that the cascade performance such as deflection angle, losses etc., is different from that of a system having no interference. In order to calculate the interference effect on cascade

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performance, a method must be found for computing the small change of flow field corresponding to the change of relative position between the two blade rows.

For this purpose the author develops in this paper a theoretical method for determining the two-dimensional incompressible potential flow through an arbitrary tandem compressor cascade based on a singularity method. The method in this paper is especially convenient for the case in which the relative position between the two blade rows varies, while the blade profiles and settings remain constant.

As previously mentioned, this theory is concerned with the solution for the flow through a given tandem compressor cascade, namely the direct problem of a tandem cascade. This problem was attacked by W. E. Spraglin⁽²⁾, K. Nickel⁽³⁾ and P. F. Byrd and M. F. Muggins⁽⁴⁾ by using the method of conformal transformation. On the other side S. Müftüoglu⁽⁵⁾ solved this problem based on a singularity method. He extended the singularity method for a single row cascade developed by H. Schlichting⁽⁷⁾, to the problem of a tandem cascade. S. Abe also proposed a singularity method⁽⁶⁾, in which he introduced an appropriate approximation for the representation of

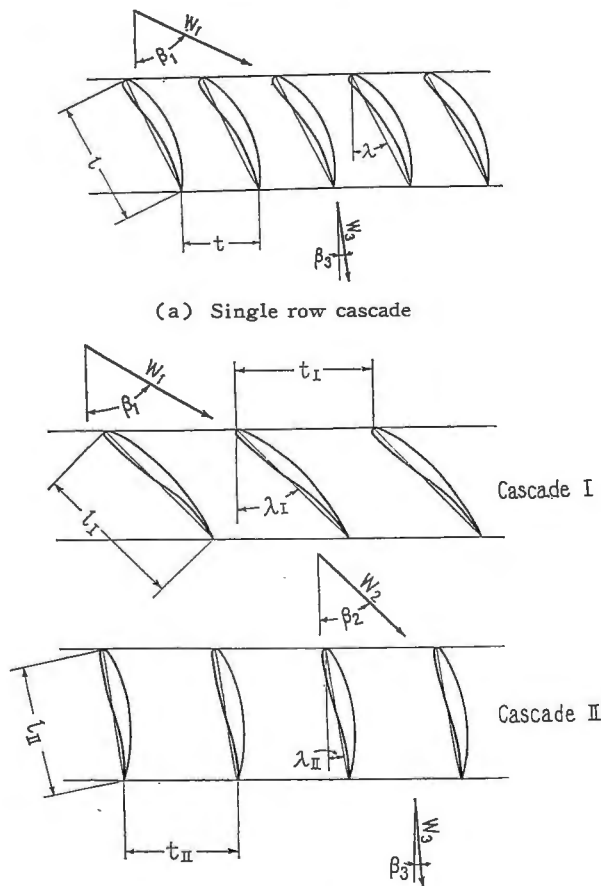
blades, and succeeded in simplifying the calculation procedure.

Since the method presented in this paper is also based on the singularity method of H. Schlichting, its basic theoretical construction is essentially similar to that of S. Müftüoglu's. However, the latter cannot be practically applied for the calculation of interference effect, because the calculation must be carried out again from the start for every change of the relative position and it takes unreasonable time to execute such calculation.

In S. Abe's method blades in cascade are represented simply only by the equivalent vorticity distribution ignoring the source/sink distribution, so that the calculation procedure is essentially easy. However, the velocity induced by the source/sink distribution has the same order of magnitude as the velocity induced by the vorticity distribution, and the accuracy of the obtained solution is not sufficient for our purpose of evaluating the small change of flow due to mutual interference.

2. Nomenclature

- l : blade chord length
 - t : blade spacing
 - x, y : co-ordinates parallel and normal to the blade chord
 - y_s : ordinate of the camber line
 - y_d : thickness distribution
 - λ : stagger angle of blades
 - W : flow velocity
 - β : flow angle relative to the cascade axis
 - u, v : induced velocity in x and y direction
 - γ, q : vorticity and source/sink distribution
- Subscripts
- 1, 3: plane upstream infinitely and downstream infinitely from tandem cascade
 - 2: plane between two blade rows of tandem cascade
 - 0: without mutual interference
 - I, II: the first and the second blade row
 - u, a : component normal and parallel to the cascade axis
 - γ, q : induced by γ and q distribution



(b) Tandem cascade

Fig. 1

3. Co-ordinate system and description of blade profile

For the calculation of two-dimensional incompressible potential flow, the geometry of a tandem compressor cascade is given as it is in Fig. 2. Both the first and the second cascades consist of weakly cambered compressor blades (max. camber/chord ≤ 0.15) with arbitrary chord length and stagger angle. However, the blade spacing of both cascades is

selected to be equal, so that the relative position between the two blade rows does not vary in the cascade direction.

Here, we select a blade from each cascade and locate origins O_I and O_{II} at the center of the two selected chord lines. Furthermore, x and y axes are set parallel and normal, respectively, to the chord lines. Hereafter, the first cascade (cascade I) is indicated by the co-ordinate system with origin O_I , the second cascade (cascade II) by the co-ordinate system with origin O_{II} .

The complex co-ordinates of both systems

$$z_I = x_I + iy_I \dots\dots\dots(1)$$

$$z_{II} = x_{II} + iy_{II} \dots\dots\dots(2)$$

are correlated by the equation

$$z_I = z_0 + z_{II} e^{-i\delta} \dots\dots\dots(3)$$

where $\delta = \lambda_I - \lambda_{II}$, and z_0 means the complex vector of origin O_{II} indicated by O_I -coordinate system; z_0 has components h and b , normal and parallel to the cascade axis, and h shows the relative shift in the cascade direction between the two blade rows, while the axial distance b may be related to the axial gap a between the two blade rows according to the equation

$$a = b - \frac{1}{2} (l_I \cos \lambda_I + l_{II} \cos \lambda_{II}) \dots\dots\dots(4)$$

Distribution of camber $y_s(x)$ and thickness $y_d(x)$ are determined by the ordinates of the upper and lower blade surface $y_0(x)$ and $y_u(x)$, according to the following equations;

$$y_s(x) = \frac{1}{2} [y_0(x) + y_u(x)] \dots\dots\dots(5)$$

$$y_d(x) = \frac{1}{2} [y_0(x) - y_u(x)] \dots\dots\dots(6)$$

As illustrated in Fig. 3, the vector mean of the inlet and outlet velocities of each blade row, is denoted by W_∞ , and has components U_∞ and V_∞

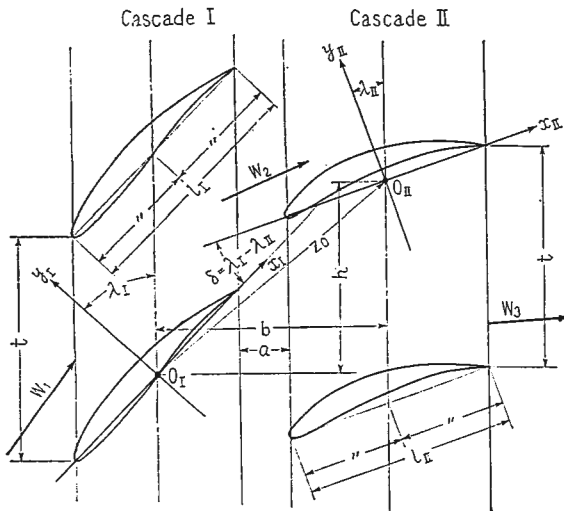


Fig. 2 Co-ordinate system of both cascades

in x and y direction.

4. Potential flow without mutual interference

The influence of mutual interference on cascade performance can be understood most clearly, when we compare the cascade performance under the influence of interference with one without interference.

Therefore, before we take up the interference problem, it is necessary to solve the potential flow through a tandem cascade without interference, and to know its cascade performance. When there is no interference, in other words when the two blade rows are located with infinite distance, a tandem cascade can be considered as a simple combination of two independent blade rows, that is, cascade I operates with inlet angle β_1 , while cascade II operates with inlet angle β_{20} , which is identical to the outlet angle of cascade I as shown in Fig. 3. In this case the flow through a tandem cascade can be obtained by solving the flow through each blade row independently and superimposing the two flow fields.

Since it is necessary to solve the flow through a single row cascade before we take up the interference problem, it is very desirable that the calculation procedure for a single row cascade is as closely as possible connected with the calculation procedure for interference effect, so that constants and coefficients can be commonly utilized for the numerical calculation of both procedures.

From the above consideration, this theory is developed in close connection with the singularity method of H. Schlichting for a single row cascade. It seems, therefore, necessary to describe hereafter some fundamentals of Schlichting's theory, which will be used directly in the following sections of this paper:

According to the singularity method, a given blade profile is generated by a closed stream line, which is obtained by superimposing the induced

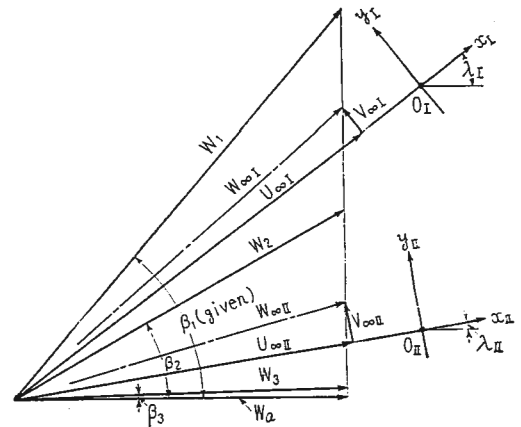


Fig. 3 Velocity triangle for both cascades

velocity field u, v on the translation flow of W_∞ .

The induced velocities u, v are caused by the vorticity and source/sink distribution on all the blade chords, and should satisfy the following two blade boundary conditions at every point on the blade chord.

Condition to produce given camber distribution;

$$\frac{V_\infty}{U_\infty} + \frac{v}{U_\infty} = y_s' \left(1 + \frac{u}{U_\infty} \right) \dots\dots\dots(7)$$

where $y_s' = \frac{dy_s}{dx}$.

Condition to produce given thickness distribution;

$$\frac{1}{2} \frac{q(x)}{U_\infty} = y_d' \left(1 + \frac{u}{U_\infty} \right) \dots\dots\dots(8)$$

where $y_d' = \frac{dy_d}{dx}$.

$$w_E = (u_{E_T} + u_{E_q}) - i(v_{E_T} + v_{E_q}) = \frac{1}{2\pi} \int_{-l/2}^{l/2} \frac{q(x') + i\gamma(x')}{x-x'} dx' \dots\dots\dots(10)$$

$$w_G = (u_{G_T} + u_{G_q}) - i(v_{G_T} + v_{G_q}) = \frac{1}{2t} \int_{-l/2}^{l/2} [q(x') + i\gamma(x')] F(x-x') dx' \dots\dots\dots(11)$$

where

$$F(x-x') = e^{i\lambda} \coth \left(\pi \frac{x-x'}{t} e^{i\lambda} \right) - \frac{1}{\pi} \frac{t}{x-x'} = R + iI \dots\dots\dots(12)$$

With the intention of simplifying the integration of Eqs. (10) and (11), the vorticity and source/sink distribution $\gamma(x), q(x)$ are represented in the form of Fourier expansion

$$\gamma(x) = 2U_\infty \left(A_0 \cot \frac{\varphi}{2} + A_1 \sin \varphi + A_2 \sin 2\varphi \right) \dots\dots\dots(13)$$

$$q(x) = 2U_\infty \left[B_0 \left(\cot \frac{\varphi}{2} - 2 \sin \varphi \right) + B_2 \sin 2\varphi + B_3 \sin 3\varphi \right] \dots\dots\dots(14)$$

where $\varphi = \cos^{-1} \left(-\frac{2x}{l} \right), \quad 0 \leq \varphi \leq \pi$

If we insert above two equations into Eqs. (10) and (11), and integrate them, we may obtain all the components of induced velocity in terms of Fourier coefficients.

The total induced velocities u and v to satisfy the conditions of Eqs. (7) and (8), are calculated by summing up all these components of induced velocity. It must be noticed here, however, that the components u_{E_T} and v_{E_q} have opposite sign on the upper and lower side of the blade chord, and therefore they are equal to zero just on the blade chord.

From the above consideration u and v are written

$$u = u_{E_q} + u_{G_T} + u_{G_q} = U_\infty [(A_0 g_{T0} + A_1 g_{T1} + A_2 g_{T2}) + (B_0 g_{q0}^* + B_2 g_{q2}^* + B_3 g_{q3}^*)] \dots\dots\dots(15)$$

$$v = v_{E_T} + v_{G_T} + v_{G_q} = U_\infty [(A_0 f_{T0}^* + A_1 f_{T1}^* + A_2 f_{T2}^*) + (B_0 f_{q0} + B_2 f_{q2} + B_3 f_{q3})] \dots\dots\dots(16)$$

where

$$\begin{aligned} g_{T0} &= -\frac{1}{t} \int \cot \frac{\varphi'}{2} Idx', & f_{T0}^* &= -\frac{1}{t} \int \cot \frac{\varphi'}{2} Rdx' - 1, \\ g_{T1} &= -\frac{1}{t} \int \sin \varphi' Idx', & f_{T1}^* &= -\frac{1}{t} \int \sin \varphi' Rdx' + \cos \varphi, \\ g_{T2} &= -\frac{1}{t} \int \sin 2\varphi' Idx', & f_{T2}^* &= -\frac{1}{t} \int \sin 2\varphi' Rdx' + \cos 2\varphi, \end{aligned}$$

(region for the above integrals is $-\frac{l}{2} \leq x' \leq \frac{l}{2}$)

$$g_{q0}^* = -f_{T0}^* + 2f_{T1}^*, \quad g_{q2}^* = -f_{T2}^*, \quad g_{q3}^* = -f_{T3}^*, \quad f_{q0} = g_{T0} - 2g_{T1}, \quad f_{q2} = g_{T2}, \quad f_{q3} = g_{T3}$$

These functions are called down-wash function and specified by three parameters $\lambda, t/l$ and x/l . The values of these functions are already tabulated in the Appendix of Reference (7), and can be easily read out.

By substituting Eqs. (15) and (16) into the boundary conditions (7) and (8), we obtain the following linear equations

The complex induced velocity $w = u - iv$ at a point on the blade chord is divided into three terms, that is

$$w = w_E + w_G + w_J \dots\dots\dots(9)$$

where

- w_E is induced by the singularity distribution on a blade chord, on which that point is located,
- w_G is induced by the singularity distribution on the remainder blade chords of the cascade, to which that point belongs, and
- w_J is induced by the singularity distribution on all the blade chords of the other cascade.

When there is no interference, w_J disappears everywhere, namely $w_J \equiv 0$.

Using the fundamental relation in the classical thin airfoil theory, we may write

$$A_0P_0 + A_1P_1 + A_2P_2 + B_0R_0 + B_2R_2 + B_3R_3 = -K + y_s' \dots\dots\dots(17\cdot a)$$

$$A_0S_0 + A_1S_1 + A_2S_2 - B_0Q_0 - B_2Q_2 - B_3Q_3 = -y_d' \dots\dots\dots(17\cdot b)$$

where parameter $K = V_\infty/U_\infty$ indicates the direction of translation flow W_∞ to the blade chord, and coefficients

$$P_i = f_{\tau_i}^* - y_s' g_{\tau_i}, \quad R_i = f_{q_i} - y_s' g_{q_i}^*, \quad S_i = y_d' g_{\tau_i}$$

$$Q_0 = \cot \frac{\varphi}{2} - 2 \sin \varphi - y_d' g_{q_0}^*$$

$$Q_2 = \sin 2\varphi - y_d' g_{q_2}^*$$

$$Q_3 = \sin 3\varphi - y_d' g_{q_3}^*$$

can be computed from the geometry of a given cascade.

Strictly speaking, Eqs. (17·a) and (17·b) must be satisfied at every point on the blade chord. However, since we represent γ and q distribution simply using only three Fourier coefficients as in Eqs. (13) and (14), the above equations can be satisfied at three of the chord positions, which are

$$\begin{cases} A_{00}P_0 + A_{10}P_1 + A_{20}P_2 + B_{00}R_0 + B_{20}R_2 + B_{30}R_3 = y_s' & \dots\dots\dots(18\cdot a) \\ A_{00}S_0 + A_{10}S_1 + A_{20}S_2 - B_{00}Q_0 - B_{20}Q_2 - B_{30}Q_3 = -y_d' & \dots\dots\dots(18\cdot b) \end{cases}$$

and

$$\begin{cases} A_{0\beta}P_0 + A_{1\beta}P_1 + A_{2\beta}P_2 + B_{0\beta}R_0 + B_{2\beta}R_2 + B_{3\beta}R_3 = -1 & \dots\dots\dots(19\cdot a) \\ A_{0\beta}S_0 + A_{1\beta}S_1 + A_{2\beta}S_2 - B_{0\beta}Q_0 - B_{2\beta}Q_2 - B_{3\beta}Q_3 = 0 & \dots\dots\dots(19\cdot b) \end{cases}$$

Finally, inlet angle β_1 , outlet angle β_2 and circulation Γ about a blade are calculated according to the equations

$$\tan \beta_1 = \frac{\tan \lambda + K + (\Delta W_u/U_\infty) \sec \lambda}{1 - K \tan \lambda}, \quad \tan \beta_2 = \frac{\tan \lambda + K - (\Delta W_u/U_\infty) \sec \lambda}{1 - K \tan \lambda} \dots\dots\dots(20), \quad (21)$$

$$\Delta W_u/U_\infty = \frac{1}{2} \frac{\Gamma}{U_\infty t} = \frac{\pi}{2} \frac{l}{t} \left[\left(A_{00} + \frac{1}{2} A_{10} \right) + K \left(A_{0\beta} + \frac{1}{2} A_{1\beta} \right) \right] \dots\dots\dots(22)$$

The method explained in this Section can be applied independently for each blade row: We calculate, at first, the flow through cascade I with a given inlet angle β_1 and determine the intermediate outlet angle β_{20} . Successively, we calculate the flow through cascade II with inlet angle β_{20} and determine the final outlet angle β_{30} . If we combine both flow fields, we get the potential flow through the given tandem cascade without mutual interference.

5. Induced velocity from the other blade row

When the distance between the two blade rows of a tandem cascade is finite, we must take account of the induced velocity w_J from the other blade row as well as $w_E + w_G$ given by Eqs. (10) and (11). In this case the γ and q distribution vary from the values γ_0 and q_0 without mutual interference, so that the total induced velocity $w = w_E + w_G + w_J$ satisfies the blade boundary conditions of Eqs. (7) and (8).

It is, therefore, seen that the first step for the evaluation of interference effect is to determine the induced velocity w_J from the other blade row.

Now, the induced velocity at a point $z = x + iy$ from the singularity distribution of a blade row with a stagger angle of λ , may be related according to the following equation;

$$w(z) = u(z) - iv(z) = \frac{e^{i\lambda}}{2t} \int_{-l/2}^{l/2} [q(x') + i\gamma(x')] \coth \left(\pi \frac{z - x'}{t} e^{i\lambda} \right) dx' \dots\dots\dots(23)$$

On the plane up- and downstream infinitely, i. e. at $z = \pm \infty$, the induced velocity may be written

$$w(\pm \infty) = \frac{e^{i\lambda}}{2t} \int_{-l/2}^{l/2} [q_0(x') + i\gamma_0(x')] \coth(\pm \infty) dx' = \pm \frac{ie^{i\lambda}}{2t} \Gamma_0$$

If we subtract above equation from Eq. (23), we obtain the change of induced velocity resulted from the existence of the other blade row, that is, w_J .

Consequently, the induced velocity w_{JII} on the blade chord of cascade II caused by the singularity-

located preferably according to the three-quarter chord method. By substituting Eqs. (13) and (14) into Eqs. (17·a) and (17·b) at these three positions, we obtain six linear simultaneous equations, and can determine from them six unknown Fourier coefficients A_0, \dots, B_3 .

The above procedure for the determination of Fourier coefficients is, however, still not satisfactory for our purpose, because the simultaneous equations must be newly solved each time for every change of attack angle $\alpha_\infty = \tan^{-1}(V_\infty/U_\infty) = \tan^{-1}K$. In order to eliminate this defect we divide Fourier coefficients into two parts and define by the equations

$$A_i = A_{i0} + KA_{i\beta}, \quad B_i = B_{i0} + KB_{i\beta}$$

If the above is inserted into Eqs. (17·a) and (17·b), we obtain two systems of simultaneous equations, which are free from K , and the divided Fourier coefficients can be determined commonly for every attack angle.

Namely,

distribution of cascade I, can be written, at first, using 0_I -coordinate system

$$w_{J_I}(z_I) = \frac{e^{i\lambda_I}}{2t} \int_{-l_{I/2}}^{l_{I/2}} [q_I(x_I) + i\gamma_I(x_I)] \coth\left(\pi \frac{z_I - x_I}{t} e^{i\lambda_I}\right) dx_I - \frac{ie^{i\lambda_I}}{2t} \Gamma_{I0} \dots\dots\dots(24)$$

Next, by means of conversion formulae

$$z_I = z_0 + z_{II} e^{-i\delta} = (b + ih)e^{-i\lambda_I} + z_{II} e^{-i\delta}, \quad w(z_I) = w(z_{II}) e^{i\delta}$$

Eq. (24) may be expressed based on 0_{II} -coordinate system

$$w_{J_{II}}(x_{II}) = \frac{e^{i\lambda_{II}}}{2t} \int_{-l_{II/2}}^{l_{II/2}} [q_{II}(x_{II}) + i\gamma_{II}(x_{II})] \coth\left[\frac{\pi}{t}(b + ih + x_{II} e^{i\lambda_{II}} - x_I e^{i\lambda_I})\right] dx_{II} - \frac{ie^{i\lambda_{II}}}{2t} \Gamma_{II0} \dots\dots\dots(25)$$

In the same way, the induced velocity w_{J_I} on the blade chord of cascade I caused by the singularity distribution of cascade II, is written, at first, using 0_{II} -coordinate system

$$w_{J_I}(z_I) = \frac{e^{i\lambda_{II}}}{2t} \int_{-l_{II/2}}^{l_{II/2}} [q_{II}(x_{II}) + i\gamma_{II}(x_{II})] \coth\left(\pi \frac{z_I - x_{II}}{t} e^{i\lambda_{II}}\right) dx_{II} + \frac{ie^{i\lambda_{II}}}{2t} \Gamma_{II0}$$

and the expression based on 0_I -coordinate system is

$$w_{J_I}(x_I) = -\frac{e^{i\lambda_{II}}}{2t} \int_{-l_{II/2}}^{l_{II/2}} [q_{II}(x_{II}) + i\gamma_{II}(x_{II})] \coth\left[\frac{\pi}{t}(b + ih + x_{II} e^{i\lambda_{II}} - x_I e^{i\lambda_I})\right] dx_{II} + \frac{ie^{i\lambda_{II}}}{2t} \Gamma_{II0} \dots\dots\dots(26)$$

The integrand of Eqs. (25) and (26) includes commonly a hyperbolic function \coth , which is specified by six independent variables b/t , h/t , x_I/t , x_{II}/t , λ_I and λ_{II} , and remains regular in the whole integral region along the blade chord. Now, we define this hyperbolic term as influence function of tandem cascade F_J , and write

$$F_J = R_J + iI_J = \coth\left[\frac{\pi}{t}(b + ih + x_{II} e^{i\lambda_{II}} - x_I e^{i\lambda_I})\right] \dots\dots\dots(27)$$

where

$$R_J = \frac{\sinh 2X}{\cosh 2X - \cos 2Y}, \quad I_J = \frac{-\sin 2Y}{\cosh 2X - \cos 2Y},$$

$$X = \frac{\pi}{t}(b + x_{II} \cos \lambda_{II} - x_I \cos \lambda_I), \quad Y = \frac{\pi}{t}(h + x_{II} \sin \lambda_{II} - x_I \sin \lambda_I)$$

If we insert this newly defined influence function into Eqs. (25) and (26), every component of w_J , such as $w_{J_I} = (u_{J_{Ir}} + u_{J_{Iq}}) - i(v_{J_{Ir}} + v_{J_{Iq}})$, $w_{J_{II}} = (u_{J_{IIr}} + u_{J_{IIq}}) - i(v_{J_{IIr}} + v_{J_{IIq}})$ can be determined by the following equations:

For cascade I;

$$u_{J_{Ir}} = \frac{\sin \lambda_I}{2t} \int_{II} \gamma_{II}(x_{II}) R_J dx_{II} + \frac{\cos \lambda_I}{2t} \int_{II} \gamma_{II}(x_{II}) I_J dx_{II} - \frac{\sin \lambda_I}{2t} \Gamma_{II0} \dots\dots\dots(29.a)$$

$$u_{J_{Iq}} = \frac{\sin \lambda_I}{2t} \int_{II} q_{II}(x_{II}) I_J dx_{II} - \frac{\cos \lambda_I}{2t} \int_{II} q_{II}(x_{II}) R_J dx_{II} \dots\dots\dots(29.b)$$

$$v_{J_{Ir}} = -\frac{\sin \lambda_I}{2t} \int_{II} \gamma_{II}(x_{II}) I_J dx_{II} + \frac{\cos \lambda_I}{2t} \int_{II} \gamma_{II}(x_{II}) R_J dx_{II} - \frac{\cos \lambda_I}{2t} \Gamma_{II0} \dots\dots\dots(29.c)$$

$$v_{J_{Iq}} = \frac{\sin \lambda_I}{2t} \int_{II} q_{II}(x_{II}) R_J dx_{II} + \frac{\cos \lambda_I}{2t} \int_{II} q_{II}(x_{II}) I_J dx_{II} \dots\dots\dots(29.d)$$

For cascade II;

$$u_{J_{IIr}} = -\frac{\sin \lambda_{II}}{2t} \int_I \gamma_I(x_I) R_J dx_I - \frac{\cos \lambda_{II}}{2t} \int_I \gamma_I(x_I) I_J dx_I + \frac{\sin \lambda_{II}}{2t} \Gamma_{I0} \dots\dots\dots(30.a)$$

$$u_{J_{IIq}} = -\frac{\sin \lambda_{II}}{2t} \int_I q_I(x_I) I_J dx_I + \frac{\cos \lambda_{II}}{2t} \int_I q_I(x_I) R_J dx_I \dots\dots\dots(30.b)$$

$$v_{J_{IIr}} = \frac{\sin \lambda_{II}}{2t} \int_I \gamma_I(x_I) I_J dx_I - \frac{\cos \lambda_{II}}{2t} \int_I \gamma_I(x_I) R_J dx_I + \frac{\cos \lambda_{II}}{2t} \Gamma_{I0} \dots\dots\dots(30.c)$$

$$v_{J_{IIq}} = -\frac{\sin \lambda_{II}}{2t} \int_I q_I(x_I) R_J dx_I - \frac{\cos \lambda_{II}}{2t} \int_I q_I(x_I) I_J dx_I \dots\dots\dots(30.d)$$

where subscripts I and II attached to the integral symbol mean the integral regions $-\frac{l_I}{2} \leq x_I \leq \frac{l_I}{2}$ and $-\frac{l_{II}}{2} \leq x_{II} \leq \frac{l_{II}}{2}$, respectively.

These integrations can be carried out numerically, or preferably by Simpson's integral formula. If we divide each blade chord of both cascades into n equal intervals for the numerical integration, the influence functions R_J and I_J in the integrands of Eqs. (29) and (30) must be evaluated for each combination of $n+1$ partition points of both cascades, in advance of integration itself.

Since these influence functions R_J and I_J contain six variables, it is almost impossible to tabulate these function values as it was done for a single row cascade.

6. Change of vorticity and source/sink distribution due to mutual interference

The vorticity and source/sink distribution, γ and q , under the influence of mutual interference are, of course, different from the value without interference, γ_0 and q_0 . If we denote the difference by γ' and q' , γ and q distribution can be expressed in the form of Fourier expansion in the same way as single row cascade

$$\gamma = \gamma_0 + \gamma' = 2U_\infty \left[(A_0 + A_0') \cot \frac{\varphi}{2} + (A_1 + A_1') \sin \varphi + (A_2 + A_2') \sin 2\varphi \right] \dots\dots\dots(31)$$

$$q = q_0 + q' = 2U_\infty \left[(B_0 + B_0') \left(\cot \frac{\varphi}{2} - 2 \sin \varphi \right) + (B_2 + B_2') \sin 2\varphi + (B_3 + B_3') \sin 3\varphi \right] \dots\dots\dots(32)$$

The Fourier coefficients without prime in the above equations correspond to the value without interference, and can be determined by the method described in Section 4. On the other hand, the Fourier coefficients with prime correspond to the difference due to mutual interference, and will be determined in this Section.

Under the influence of mutual interference, the total induced velocities u and v , which should satisfy the blade boundary conditions (7) and (8), are written

$$u = u_{E_q} + u_{G_\gamma} + u_{G_q} + u_{J_\gamma} + u_{J_q}, \quad v = v_{E_\gamma} + v_{G_\gamma} + v_{G_q} + v_{J_\gamma} + v_{J_q} \dots\dots\dots(33)$$

If we substitute Eqs. (15) and (16) into the above equations, we obtain

$$\frac{u}{U_\infty} = [(A_0 + A_0')g_{r0} + (A_1 + A_1')g_{r1} + (A_2 + A_2')g_{r2} + (B_0 + B_0')g_{q0}^* + (B_2 + B_2')g_{q2}^* + (B_3 + B_3')g_{q3}^*] + \frac{uJ}{U_\infty} \dots\dots\dots(34)$$

$$\frac{v}{U_\infty} = [(A_0 + A_0')f_{r0}^* + (A_1 + A_1')f_{r1}^* + (A_2 + A_2')f_{r2}^* + (B_0 + B_0')f_{q0} + (B_2 + B_2')f_{q2} + (B_3 + B_3')f_{q3}] + \frac{vJ}{U_\infty} \dots\dots\dots(35)$$

By setting these expressions in the condition of Eq. (7) and rearranging in terms of Fourier coefficients, we obtain

$$\begin{aligned} & (A_0 + A_0') \frac{(f_{r0}^* - y_s' g_{r0})}{= P_0} + (A_1 + A_1') \frac{(f_{r1}^* - y_s' g_{r1})}{= P_1} + (A_2 + A_2') \frac{(f_{r2}^* - y_s' g_{r2})}{= P_2} \\ & + (B_0 + B_0') \frac{(f_{q0} - y_s' g_{q0}^*)}{= R_0} + (B_2 + B_2') \frac{(f_{q2} - y_s' g_{q2}^*)}{= R_2} + (B_3 + B_3') \frac{(f_{q3} - y_s' g_{q3}^*)}{= R_3} \\ & = -K + y_s' + \frac{1}{U_\infty} (y_s' u_J - v_J) \end{aligned}$$

Since Fourier coefficients without prime $A_0, A_1, A_2, B_0, B_2, B_3$ in the above relation satisfy Eq. (17·a), we can subtract both sides of Eq. (17·a) from both sides of the above relation. Consequently, we get a linear equation in terms of unknown Fourier coefficients with prime

$$A_0' P_0 + A_1' P_1 + A_2' P_2 + B_0' R_0 + B_2' R_2 + B_3' R_3 = \frac{1}{U_\infty} (y_s' u_J - v_J) \dots\dots\dots(36)$$

where P_0, \dots, R_0, \dots have been defined for a single row cascade in Section 4, and can be used again for this calculation.

In the same way, by setting Eqs. (34) and (35) in the condition of Eq. (8) and rearranging, we obtain

$$\begin{aligned} & (A_0 + A_0') \frac{y_d' g_{r0}}{= S_0} + (A_1 + A_1') \frac{y_d' g_{r1}}{= S_1} + (A_2 + A_2') \frac{y_d' g_{r2}}{= S_2} - (B_0 + B_0') \frac{\left(\cot \frac{\varphi}{2} - 2 \sin \varphi - y_d' g_{q0}^* \right)}{= Q_0} \\ & - (B_2 + B_2') \frac{(\sin 2\varphi - y_d' g_{q2}^*)}{= Q_2} - (B_3 + B_3') \frac{(\sin 3\varphi - y_d' g_{q3}^*)}{= Q_3} = -y_d' - y_d' \frac{u_J}{U_\infty} \end{aligned}$$

By subtracting both sides of Eq. (17·b) from both sides of the above relation, we get the other linear equation

$$A_0' S_0 + A_1' S_1 + A_2' S_2 - B_0' Q_0 - B_2' Q_2 - B_3' Q_3 = -y_d' \frac{u_J}{U_\infty} \dots\dots\dots(37)$$

Here, we select three points on the blade chord of cascade I and II according to the three-quarter chord method, and indicate these points with $\mu = 1, 2, 3$ and $\nu = 1, 2, 3$, respectively. If the induced velocities from the other blade row, u_J and v_J , are supposed to be known at these selected points, Fourier coefficients with

prime can be determined by solving the following simultaneous equations:

For cascade I ;

$$\left. \begin{aligned} A_0' P_{0\mu} + A_1' P_{1\mu} + A_2' P_{2\mu} + B_0' R_{0\mu} + B_2' R_{2\mu} + B_3' R_{3\mu} &= \frac{1}{U_{\infty I}} (y_{\delta\mu}' u_{J I} - v_{J I}) \\ A_0' S_{0\mu} + A_1' S_{1\mu} + A_2' S_{2\mu} - B_0' Q_{0\mu} - B_2' Q_{2\mu} - B_3' Q_{3\mu} &= -\frac{1}{U_{\infty I}} y_{\delta\mu}' u_{J I} \end{aligned} \right\} \mu=1, 2, 3 \dots\dots\dots(38)$$

For cascade II ;

$$\left. \begin{aligned} A_0' P_{0\nu} + A_1' P_{1\nu} + A_2' P_{2\nu} + B_0' R_{0\nu} + B_2' R_{2\nu} + B_3' R_{3\nu} &= \frac{1}{U_{\infty II}} (y_{\delta\nu}' u_{J II} - v_{J II}) \\ A_0' S_{0\nu} + A_1' S_{1\nu} + A_2' S_{2\nu} - B_0' Q_{0\nu} - B_2' Q_{2\nu} - B_3' Q_{3\nu} &= -\frac{1}{U_{\infty II}} y_{\delta\nu}' u_{J II} \end{aligned} \right\} \nu=1, 2, 3 \dots\dots\dots(39)$$

As explained in the previous Section, $u_{J I}$ and $v_{J I}$ are determined by the unknown singularity distribution $(\gamma_0 + \gamma')$ _I and $(q_0 + q')$ _I of cascade II, in turn, $u_{J II}$ and $v_{J II}$ by the unknown singularity distribution $(\gamma_0 + \gamma')$ _I and $(q_0 + q')$ _I of cascade I, and consequently, their values are all unknown.

However, if we compare Eq. (17) with Eqs. (38) and (39), it becomes clear that the order of magnitude of γ', q' to γ_0, q_0 is equal to $|w_J|/U_\infty$, and can be assumed small. This suggests us the possibility, that u_J and v_J can be determined with sufficient accuracy by using the known singularity distribution, γ_0 and q_0 , without interference instead of by using the actual unknown singularity distribution. If we substitute these approximate values of u_J, v_J into simultaneous equations (38) and (39), the change of singularity distribution can be determined as the first approximation. In order to improve the accuracy of the solution, we must carry out, of course, iterative calculation.

Now, we consider the accuracy of the solution when we cut off the calculation by the first approximation. As illustrated in Fig. 4, γ_0 and q_0 distribution of cascade I induces $w_{J I}$ on the blade chord of cascade II, and $w_{J II}$ conditions the change of singularity distribution γ', q' of cascade II. Furthermore, this γ', q' induces $w_{J I}'$ on the blade chord of cascade I, and $w_{J I}'$ also conditions the change of singularity distribution γ'', q'' of cascade I. Therefore, if we adopt the first approximation, we have to disregard the terms smaller than γ'', q'' .

Speaking of a usual tandem compressor cascade, the maximum value of $|w_J|/U_\infty$ reaches about 10^{-2} , that is, the order of magnitude of γ', q' and γ'', q'' to γ_0, q_0 is about 10^{-2} and 10^{-4} , respectively. As a result, it is understood that the first approximation assures practically enough accuracy for our purpose, and any further iterative calculation becomes meaningless, for many assumptions adopted in this theory may cause errors, at least, comparable to γ'', q'' .

When the relative position between the two blade rows varies, and the blade profiles and settings remain constant, only the terms in the right side of Eqs. (38) and (39) vary, while the terms in the left side remain unchanged. Therefore, it is comparatively easy to determine the solutions corresponding to many relative positions.

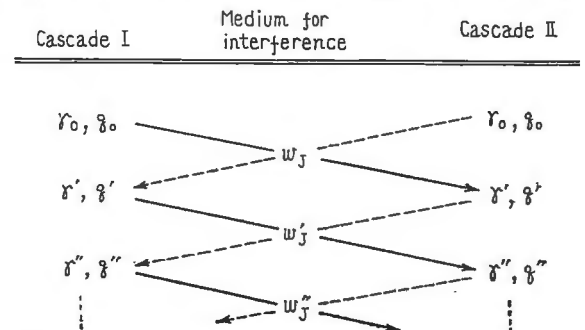


Fig. 4 Relation between the accuracy of solution and iterative procedure

7. Change of outlet angle and circulation due to mutual interference

Fourier coefficients under the influence of mutual interference are written

$$A_i = A_{i0} + A_i' + KA_{i\beta} \quad (i=0, 1, 2), \quad B_i = B_{i0} + B_i' + KB_{i\beta} \quad (i=0, 2, 3) \dots\dots\dots(40)$$

where $A_{i0}, A_{i\beta}, A_i', B_{i0}, B_{i\beta}, B_i'$ have been determined in the previous Section. However, parameter K in the above equations differs from the value K_0 without interference, and will be determined hereafter.

Circulation Γ about a blade can be obtained by integrating the vorticity distribution on the blade chord

$$\Gamma = \int_{-l/2}^{l/2} \gamma(x) dx = \pi U_\infty l \left[\left(A_{00} + \frac{1}{2} A_{10} \right) + \left(A_0' + \frac{1}{2} A_1' \right) + K \left(A_{0\beta} + \frac{1}{2} A_{1\beta} \right) \right] \dots\dots\dots(41)$$

Besides, lift coefficients C_L and down-wash velocity ΔW_u are written

$$C_L = \rho \Gamma W_\infty / \frac{\rho}{2} W_\infty^2 l = 2\pi \left[\left(A_{00} + \frac{1}{2} A_{10} \right) + \left(A_0' + \frac{1}{2} A_1' \right) + K \left(A_{0\beta} + \frac{1}{2} A_{1\beta} \right) \right] \cos \alpha_\infty$$

$$= 2\pi \left\{ \left[\left(A_{00} + \frac{1}{2} A_{10} \right) + \left(A_0' + \frac{1}{2} A_1' \right) \right] \cos \alpha_\infty + \left(A_{0\beta} + \frac{1}{2} A_{1\beta} \right) \sin \alpha_\infty \right\} \dots\dots\dots(42)$$

$$\frac{\Delta W_u}{U_\infty} = \frac{1}{2} \frac{\Gamma}{U_\infty t} = \frac{\pi}{2} \frac{l}{t} \left[\left(A_{00} + \frac{1}{2} A_{10} \right) + \left(A_0' + \frac{1}{2} A_1' \right) + K \left(A_{0\beta} + \frac{1}{2} A_{1\beta} \right) \right] \dots\dots\dots(43)$$

Substituting Eq. (43) into Eq. (20), K_I of cascade I for a given inlet angle β_1 may be related according to the equation

$$K_I = \frac{\tan \beta_1 - \tan \lambda_I - \sec \lambda_I \frac{\pi}{2} \frac{l_I}{t} \left[\left(A_{00} + \frac{1}{2} A_{10} \right) + \left(A_0' + \frac{1}{2} A_1' \right) \right]_I}{1 + \tan \lambda_I \tan \beta_1 + \sec \lambda_I \frac{\pi}{2} \frac{l_I}{t} \left(A_{0\beta} + \frac{1}{2} A_{1\beta} \right)_I} \dots\dots\dots(44)$$

Using K_I determined by the above equation, singularity distribution, circulation, lift coefficient and down-wash velocity of cascade I can be calculated by Eqs. (40) to (43), where the normalizing velocity $U_{\infty I}$ is written

$$U_{\infty I} = W_a \sec(\lambda_I + \tan^{-1} K_I) \cos(\tan^{-1} K_I) \dots\dots\dots(45)$$

The outlet angle β_2 of cascade I, which does not indicate the actual local flow angle between the two blade rows, but indicates the mean flow angle, is calculated by the equation

$$\tan \beta_2 = \frac{\tan \lambda_I + K_I - \sec \lambda_I (\Delta W_{uI} / U_{\infty I})}{1 - K_I \tan \lambda_I} \dots\dots\dots(46)$$

With the understanding that this β_2 becomes the inlet angle of cascade II, we can write further equations

$$K_{II} = \frac{\tan \beta_2 - \tan \lambda_{II} - \sec \lambda_{II} \frac{\pi}{2} \frac{l_{II}}{t} \left[\left(A_{00} + \frac{1}{2} A_{10} \right) + \left(A_0' + \frac{1}{2} A_1' \right) \right]_{II}}{1 + \tan \lambda_{II} \tan \beta_2 + \sec \lambda_{II} \frac{\pi}{2} \frac{l_{II}}{t} \left(A_{0\beta} + \frac{1}{2} A_{1\beta} \right)_{II}} \dots\dots\dots(47)$$

$$U_{\infty II} = W_a \sec(\lambda_{II} + \tan^{-1} K_{II}) \cos(\tan^{-1} K_{II}) \dots\dots\dots(48)$$

$$\tan \beta_3 = \frac{\tan \lambda_{II} + K_{II} - \sec \lambda_{II} (\Delta W_{uII} / U_{\infty II})}{1 - K_{II} \tan \lambda_{II}} \dots\dots\dots(49)$$

8. Change of pressure distribution and losses due to mutual interference

From the obtained singularity distribution the velocity distribution along the blade surface can be calculated. For this purpose the method for a single row cascade⁽⁷⁾⁽⁸⁾ can be applied only with the modification that the contribution of the induced velocity u_J, v_J from the other blade row is also taken into account.

We consider the flow velocity at point A on the camber line as illustrated in Fig. 5. From the blade boundary condition of Eq. (7) the resultant velocity W_t of $U_\infty + u = U_\infty + u_{E_q} + u_G + u_J$ and $V_\infty + v = V_\infty + v_{E_r} + v_G + v_J$, is parallel to the tangent of the camber line, and the magnitude is

$$W_t = (U_\infty + u_{E_q} + u_G + u_J) \cos \theta + (V_\infty + v_{E_r} + v_G + v_J) \sin \theta$$

$$= \frac{1}{\sqrt{1 + y_s'^2}} \left[(U_\infty + u_{E_q} + u_G + u_J) + y_s' (V_\infty + v_{E_r} + v_G + v_J) \right] \dots\dots\dots(50)$$

In order to calculate the velocity on the upper and lower side of the camber line, we must also consider the contributions of u_{E_r} and v_{E_q} , which disappear just on the blade chord.

In this paper it has been previously assumed that vorticity and source/sink may be distributed with sufficient accuracy on the blade chord (x -axis) instead of on the camber line (s -axis), so long as we limit our problem to a weakly cambered blade profile. However, this assumption becomes inconsistent only for the calculation of u_{E_r} and v_{E_q} , and in this case we need to put the vorticity and source/sink distribution exactly on the camber line.

The singularity distributions on the camber line $\gamma(s), q(s)$ are related to $\gamma(x), q(x)$ on the blade chord according to the assumption that the singularity included in dx is equal to that included in ds , that is

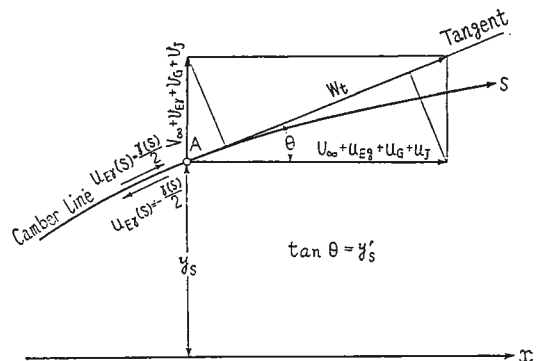


Fig. 5 Velocity distribution along camber line

$$\left. \begin{aligned} \gamma(s) &= \gamma(x) \frac{dx}{ds} = \gamma(x) \cos \theta = \frac{\gamma(x)}{\sqrt{1+y_s'^2}} \\ q(s) &= q(x) \frac{dx}{ds} = q(x) \cos \theta = \frac{q(x)}{\sqrt{1+y_s'^2}} \end{aligned} \right\} \dots(51)$$

The source/sink distribution $q(s)$ on the camber line induces $v_{E_q} = \pm q(s)/2$ normal to the camber line on the upper (sign +) and lower (sign -) side, and it gives no effect on the velocity parallel to the camber line.

On the other hand, the vorticity distribution $\gamma(s)$ induces $u_{E_\gamma}(s) = \pm \frac{\gamma(s)}{2} = \pm \frac{1}{\sqrt{1+y_s'^2}} \frac{\gamma(x)}{2}$ tangential to the camber line, and the velocity W_s on the upper and lower side of the camber line is obtained by superimposing $v_{E_\gamma}(s)$ to W_i .

$$\begin{aligned} \frac{W_{s0}}{U_{\infty 0}} &= \frac{1}{\sqrt{1+y_s'^2}} \left[1 + A_{00}g_{r0} + A_{10}g_{r1} + A_{20}g_{r2} + B_{00}g_{q0}^* + B_{20}g_{q2}^* + B_{30}g_{q3}^* \right. \\ &\quad + K_0(A_{0\beta}g_{r0} + A_{1\beta}g_{r1} + A_{2\beta}g_{r2} + B_{0\beta}g_{q0}^* + B_{2\beta}g_{q2}^* + B_{3\beta}g_{q3}^*) \\ &\quad + y_s'(K_0 + A_{00}f_{r0}^* + A_{10}f_{r1}^* + A_{20}f_{r2}^* + B_{00}f_{q0} + B_{20}f_{q2} + B_{30}f_{q3}) \\ &\quad + y_s'K_0(A_{0\beta}f_{r0}^* + A_{1\beta}f_{r1}^* + A_{2\beta}f_{r2}^* + B_{0\beta}f_{q0} + B_{2\beta}f_{q2} + B_{3\beta}f_{q3}) \\ &\quad \left. \pm \left(A_{00} \cot \frac{\varphi}{2} + A_{10} \sin \varphi + A_{20} \sin 2\varphi \right) \pm K_0 \left(A_{0\beta} \cot \frac{\varphi}{2} + A_{1\beta} \sin \varphi + A_{2\beta} \sin 2\varphi \right) \right] \dots(55) \end{aligned}$$

If we denote the difference of Fourier coefficients between the cases with and without interference effect by using asterisk, that is

$$A_i^* = A_i' + (K - K_0)A_{i\beta} \quad (i=0, 1, 2), \quad B_i^* = B_i' + (K - K_0)B_{i\beta} \quad (i=0, 2, 3) \dots(56)$$

the variation W_s' is written in dimensionless form

$$\begin{aligned} \frac{W_s'}{U_{\infty 0}} &\cong \frac{W_s'}{U_{\infty}} = \frac{1}{\sqrt{1+y_s'^2}} \left[A_0^*g_{r0} + A_1^*g_{r1} + A_2^*g_{r2} + B_0^*g_{q0}^* + B_2^*g_{q2}^* + B_3^*g_{q3}^* + \frac{u_J}{U_{\infty}} \right. \\ &\quad + y_s' \left(A_0^*f_{r0}^* + A_1^*f_{r1}^* + A_2^*f_{r2}^* + B_0^*f_{q0} + B_2^*f_{q2} + B_3^*f_{q3} + \frac{v_J}{U_{\infty}} \right) \\ &\quad \left. \pm \left(A_0^* \cot \frac{\varphi}{2} + A_1^* \sin \varphi + A_2^* \sin 2\varphi \right) \right] \dots(57) \end{aligned}$$

Finally from the blade surface velocity distribution W_k , pressure coefficient C_p on the blade surface can be calculated

$$C_p = \frac{p_k - p_1}{\frac{\rho}{2} W_1^2} = 1 - \left(\frac{W_k}{W_1} \right)^2 = 1 - \left(\frac{W_k}{U_{\infty 0}} \right)^2 \left(\frac{U_{\infty 0}}{W_1} \right)^2 \dots(58)$$

The change of losses due to mutual interference can be determined further from the obtained blade surface velocity distribution, by executing the boundary layer calculation using, for example, the method by L. Speidel and N. Scholz⁽⁹⁾.

9. Sample calculation

Sample calculations are carried out for a tandem compressor cascade specified in Table 1, and at the relative positions between the two blade rows listed in Table 2.

Table 1 Specification of tandem cascade for sample calculation

	Cascade I	Cascade II
Blade profile	NACA 8 410	NACA 8 410
Spacing to chord ratio	0.75	0.75
Stagger angle	37.4°	8.9°
Inlet angle	50°	=outlet angle of cascade I
Reynolds No.	1.7×10 ⁵ based on W_{20}	1.5×10 ⁵ based on W_{30}

Table 2 Relative position between the two blade rows

a/l	h/l		
0.15	0.457 9	0.707 9	0.957 9
0.30	0.534 7	0.784 7	1.034 7
∞	—		

$$W_s = \frac{1}{\sqrt{1+y_s'^2}} \left[(U_{\infty} + u_{E_q} + u_G + u_J) + y_s'(V_{\infty} + v_{E_\gamma} + v_G + v_J) \pm \frac{\gamma(x)}{2} \right] \dots(52)$$

The velocity distribution W_k on the blade surface can be derived from W_s by transforming the camber line approximately to the blade profile, and is related to W_s by Riegelsfactor $1/\sqrt{1+y_d'^2}$

$$W_k = \frac{W_s}{\sqrt{1+y_d'^2}} \dots(53)$$

In order to facilitate the evaluation of W_s at various relative positions, it is better to express W_s as the sum of the velocity W_{s0} without interference and the velocity change W_s' due to interference.

$$\frac{W_s}{U_{\infty 0}} = \frac{W_{s0}}{U_{\infty 0}} + \frac{W_s'}{U_{\infty 0}} \dots(54)$$

Further, W_{s0} may be written in dimensionless form, using Eqs. (15) and (16)

Table 3 shows the result such as intermediate outlet angle β_2 , final outlet angle β_3 , relative change of circulation Γ_I/Γ_{I0} , Γ_{II}/Γ_{II0} and loss coefficient $\zeta_V = \Delta p_i / \frac{\rho}{2} W_{i0}^2$.

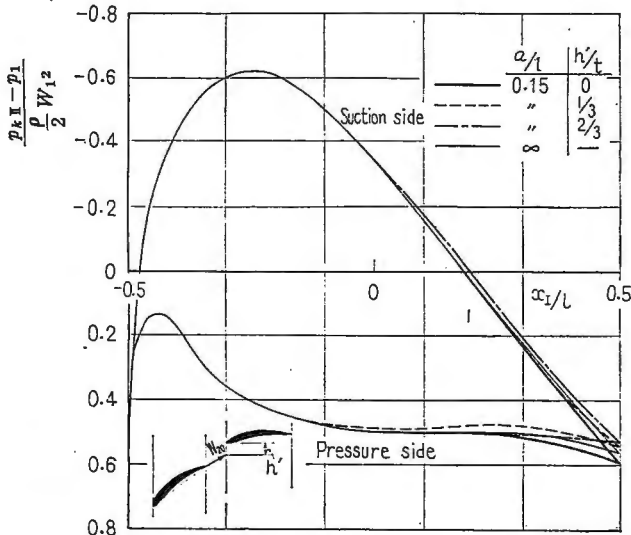


Fig. 6 Pressure distribution of cascade I

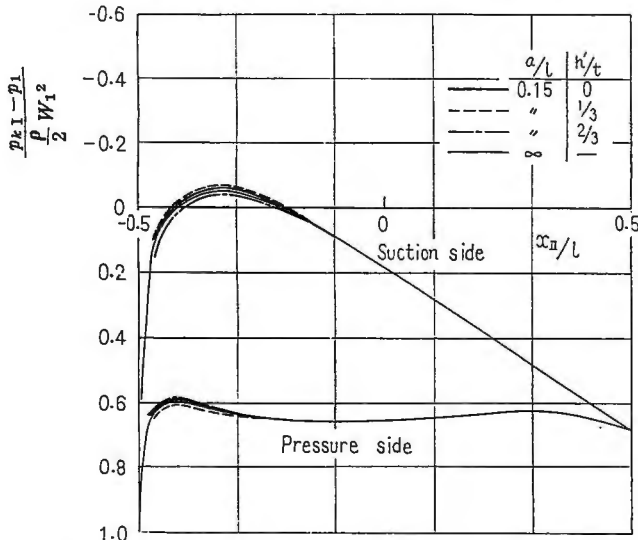


Fig. 7 Pressure distribution of cascade II

Figs. 6 and 7 illustrate the dimensionless pressure distributions of cascades I and II corresponding to various relative positions.

Fig. 8 shows the results of boundary layer calculation, and the loss coefficients of both cascades, ζ_{VI} and ζ_{VII} , are plotted against the relative shift in cascade direction between the two blade rows. For the comparison between theory and experiment, the measured loss coefficients⁽¹⁾ are also plotted.

Here, we compare the theoretical change of loss coefficients with experimental result. Concerning cascade I, as seen in Fig. 8, theory can explain experimental results comparatively well, especially at $a/l=0.15$ the amplitude as well as the phase of loss curves practically agrees with each other.

In consequence of such good agreement, it seems that the mechanism of the interference effect in

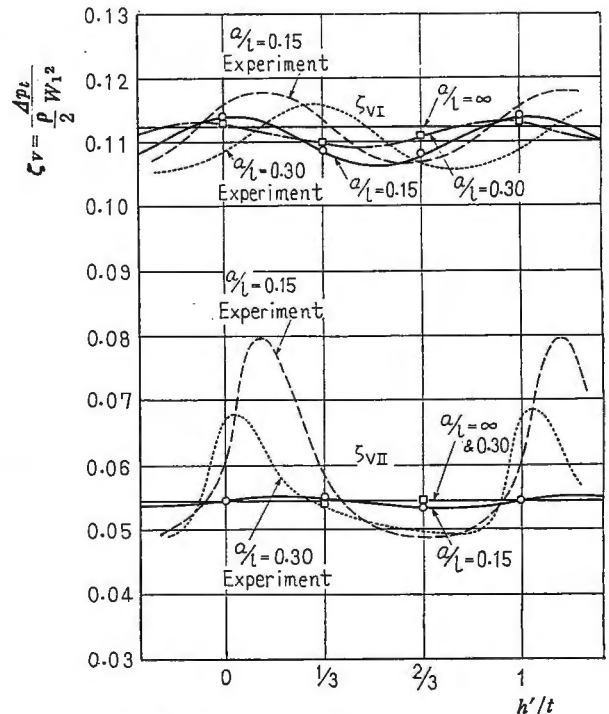


Fig. 8 Change of loss coefficient due to mutual interference

Table 3 Results of sample calculation

a/l	0.15			0.30			∞	
	h/l	0.457 9	0.707 9	0.957 9	0.534 7	0.784 7		1.034 7
h'/t		0	1/3	2/3	0	1/3	2/3	—
β_2		30°01'	30°53'	29°18'	30°12'	30°12'	29°46'	30°03'
β_3		0°14'	0°14'	0°13'	0°14'	0°14'	0°13'	0°14'
Γ_I/Γ_{I0}		1.001 8	0.963 6	1.032 8	0.993 6	0.993 6	1.012 7	1
Γ_{II}/Γ_{II0}		0.999 2	1.032 5	0.971 2	1.006 0	1.006 0	0.989 4	1
$\frac{\Gamma_I + \Gamma_{II}}{\Gamma_{I0} + \Gamma_{II0}}$		1.000 5	0.998 0	1.002 0	0.999 8	0.999 8	1.001 0	1
ζ_{VI}		0.113 5	0.108 6	0.107 8	0.113 0	0.109 6	0.110 6	0.112 3
ζ_{VII}		0.054 5	0.054 9	0.053 3	0.054 4	0.054 4	0.054 4	0.054 4
$\zeta_{VI} + \zeta_{VII}$		0.168 0	0.163 5	0.161 1	0.167 4	0.164 0	0.165 0	0.166 7

cascade I is explained essentially by means of the change of potential flow resulting from the existence of cascade II.

On the other hand, the agreement between theory and experiment about cascade II is very poor.

It is found, therefore, that the mechanism of the interference effect in cascade II might be quite different from that in cascade I. The greater part of the change of losses in cascade II might be attributable to the mutual interference between the wake from cascade I and the boundary layer of cascade II, as explained in Reference (1).

10. Conclusion

In order to explain the influence of mutual interference in a tandem compressor cascade theoretically, a theory for the two-dimensional incompressible potential flow through a tandem cascade based on a singularity method has been developed.

As the result of sample calculations, it has become clear that regarding the first cascade the change of cascade performance due to interference effect can be predicted by this theory with satisfactory agreement, while the change of cascade perform-

ance about the second cascade can not be estimated by this potential theory. Because, in the second cascade the viscous effect such as the interference between the wake and the boundary layer becomes predominant.

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