

# Lateral Fluid Forces on Whirling Centrifugal Impeller (1st Report: Theory)

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*Lateral fluid forces acting on a rotating centrifugal impeller in whirling motion are analyzed using unsteady potential flow theory. Impellers operating in diffusers with and without vanes are modeled and the fluid forces calculated for different whirl speeds and flow rates. The influences of these parameters are clarified by parametric calculations. The results for whirling impellers operating in vaneless diffusers show that the fluid forces exert a damping effect on the rotor whirling motion at all operating conditions. The results for impellers operating in vanned diffusers or guide vanes show that the time averaged values of fluid forces remain almost unchanged, while there are significant instantaneous fluctuations due to the impeller/guide vane interactions.*

## Introduction

The rotordynamic stability of the rotating shaft of high performance turbomachines is as important as its fluid dynamic performance. This is especially true for the case of high-speed multistage centrifugal pumps and compressors whose rotors are usually less rigid than in other machines. The rotordynamic behavior of such systems must be studied to enable accurate estimates of their response to unbalanced mass and to seismic loads.

It is necessary for the stability analyses of centrifugal rotors to estimate the fluid dynamic forces acting on each element of the vibrating rotor such as journal bearing, wear ring, throttle bushing, balance piston and centrifugal impeller. Among these elements, the fluid forces on journal bearings have been studied most extensively. Bearing data are available for various geometric parameters in the form of stiffness and damping constants (Someya 1987) and solutions are available for instability problems like oil whip such as observed in journal bearings. This has been accompanied by progress in the study of the fluid forces on annular pressure seals such as wear rings and throttle bushings (Black 1969, Iino 1980, Childs 1984).

Many studies have been made on the steady lateral fluid forces which act on the centrifugal impeller caused either by azimuthally asymmetric configuration of volute casing or by misalignment of impeller and guide vanes (Hergt 1969, Domm 1970). When a rotating shaft with a centrifugal impeller whirls as shown in Fig. 1, the impeller motion is the resultant of the shaft center whirl and the shaft rotation. In this case the flow around the impeller becomes unsteady and asymmetric, in spite of the fact that its casing is completely symmetric. This causes unsteady fluid forces acting on the impeller. The force component normal to the shaft is denoted as lateral force.

This component plays a predominant role in the shaft vibration in bending mode.

The present study deals with the lateral fluid forces on centrifugal impellers for the case of impeller whirling on a circular orbit with a constant eccentricity. The lateral force component is further divided into normal and parallel components relative to the whirl motion. The influence of impeller geometry, ratio of whirl speed to impeller speed, and flow condition (flow coefficient) etc. are investigated theoretically in this article and experimentally in the following article.

In this study, an unsteady incompressible potential flow theory is introduced to estimate the fluid forces on a two-dimensional centrifugal impeller whirling on a circular orbit with a constant whirl speed. This theory takes into consideration shed vortices from the impeller vanes. In the accompanying article, the 2nd Report, an experimental study of these fluid forces is presented for two-dimensional impellers whirling in vaneless diffusers along with comparisons between theory and experiment.

Previous researchers have modeled the unsteady part of lateral forces, which occur from the whirling motion of a rotating impeller or to the interaction between impeller and

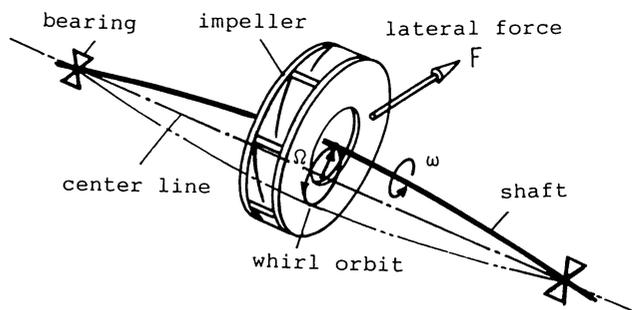


Fig. 1 Centrifugal impeller in whirling motion

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guide vanes. Chamieh (1983) applied an actuator disc model to the calculation of lateral forces on a whirling centrifugal impeller in a volute. The result of this analysis was, however, unable to explain the measured behavior of tangential force properly. This actuator disc model was later improved by Tsujimoto, Acosta and Brennen (1986) and the calculated radial and tangential forces were shown to qualitatively agree with experimental measurements.

Thompson (1978) has treated the case of an unbounded centrifugal impeller under the assumption of quasi-steady flow. The perturbed flow in each impeller passage was solved by the streamline curvature method. This calculation proved that whirl-exciting fluid forces occur for some impeller parameters. Since it is questionable whether a quasi-steady theory is realistic for this kind of highly unsteady problem, no generalizations were given.

Colding-Jorgensen (1980) treated the case of a whirling centrifugal impeller in a spiral volute casing under the assumption of quasi-steady flow. In their analysis the casing was replaced by distributed vortices, and the impeller by a point source with constant circulation. Although the theory succeeded in determining the stiffness and damping coefficients of fluid forces, the results should be carefully checked since the theory is based on a simple flow model. Allaire et al. (1984) has reported a theory using a finite element model of a full impeller.

The authors (1980) earlier treated the case of whirling centrifugal impeller in a vaneless diffuser using the singularity method for the whole flow field including shed vortices. This theory was restricted to the case of shockfree entry to the leading edge of impeller vanes. The theory presented in this paper is not restricted to shockfree entry condition. It is used to treat the case in which the impeller whirls in a vaned diffuser or in an asymmetric volute casing under the assumptions of two-dimensional, inviscid, incompressible flow.

### Whirling Motion and Lateral Force

The bending mode vibration of a rotating shaft typically appears as a whirling motion with an elliptical orbit as shown in Fig. 1. If we consider the simple case of a circular orbit, the plane motion of the impeller is illustrated in Fig. 2. The impeller rotates counterclockwise with angular speed  $\omega$  around its center  $O$ , which revolves with angular speed  $\Omega$  around its absolute center  $O'$  along a circular orbit with eccentricity  $|\epsilon|$ . Positive whirl occurs, when the shaft whirls in the same direction as its rotation, and negative whirl occurs when the shaft whirl is in the opposite direction.

When an impeller rotates without whirl in a vaneless diffuser or in the center of a vaned diffuser, the flow around the impeller becomes azimuthally symmetric and no lateral force acts on the impeller. However once a whirling motion occurs,

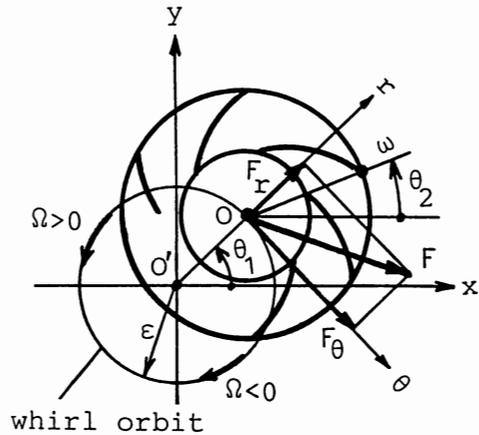


Fig. 2 Fluid forces acting on impeller

the flow becomes asymmetric and causes an unsteady lateral force  $F$ . This lateral force is further divided into radial and tangential components relative to the orbit as shown in Fig. 2. Here an outward radial force  $F_r$  is taken as positive. To avoid confusion the fluid force normal to the shaft is called lateral force instead of the commonly used radial thrust.

The radial force component  $F_r$  applies a bending moment to the shaft and influences the shaft's natural frequency in bending. The tangential force component  $F_\theta$  acts parallel to the motion of shaft center  $O$ , and thus transfers kinetic energy to the whirling rotor. In positive whirl, a positive  $F_\theta$  acts against the whirl velocity and exerts a stabilizing or damping effect, while a negative  $F_\theta$  exerts a destabilizing or excitatory effect. In negative whirl these effects are reversed. When the tangential force becomes excitatory and feeds more kinetic energy to the whirling shaft than it damps out, a severe impeller-induced self-excitation, or impeller whip occurs which is similar to oil whip.

In the absence of external forces, the bending mode frequency of whirling motion is equal to the natural frequency or critical speed of the shaft. Therefore, the case of whirl speed ratio  $|\Omega/\omega| > 1$  corresponds to subcritical, while  $|\Omega/\omega| < 1$  corresponds to supercritical rotational conditions. When the shaft experiences forced vibration due to an external force like a seismic load, the whirling frequency coincides with that of the external force. In this study the range of whirl speed ratio, covering positive to negative ratios, was selected to deal with these possible cases.

### Major Assumptions

Since the flow around a real centrifugal impeller in whirling

### Nomenclature

$b$ = vane width	$r_3$ = inner radius of guide vanes	
$F$ = fluid force vector	$r_4$ = outer radius of guide vanes	tion, positive
$f$ = nondimensional fluid force vector	$v$ = complex expression of velocity	counterclockwise
$M$ = mass of the fluid displaced by the projected volume of impeller vanes	$z$ = complex expression of position	$\omega$ = impeller angular speed of rotation, positive
$M_a$ = apparent mass of impeller calculated by potential theory	$\bar{z}$ = complex conjugate of $z$	counterclockwise
$N$ = number of vanes	$\beta$ = angle between radius and vanes	
$Q$ = flow rate	$\gamma$ = vortex distribution	<b>Subscripts</b>
$r_1$ = inner radius of impeller	$\epsilon$ = complex expression of eccentricity	$a$ = free vortices
$r_2$ = outer radius of impeller	$\rho$ = density of fluid	$G$ = guidevane
	$\phi$ = flow coefficient	$r$ = radial component
	$\Omega$ = angular speed of whirling motion, positive	$sf$ = shockfree entry condition
		$T$ = trailing edge of vane
		$\theta$ = tangential component

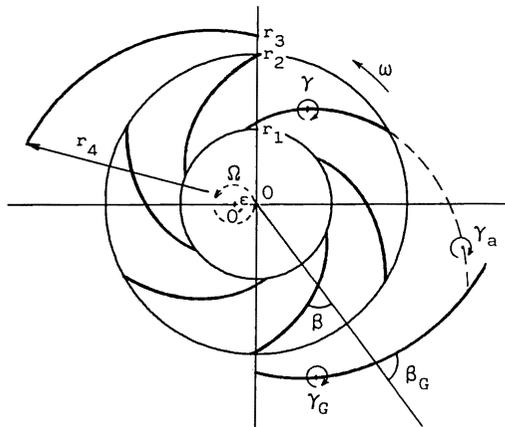


Fig. 3 Impeller surrounded by guide vanes

motion is very complex, in the present theory the following simplifications and assumptions are introduced:

- (1) The flow is assumed inviscid, incompressible, two-dimensional and irrotational in the absolute coordinate system.
- (2) Flow rate, rotational speed, eccentricity, and whirl speed of orbital motion are assumed to be constant.
- (3) The ratio of whirl eccentricity to the radius of the impeller is assumed to be very small.
- (4) The geometries of impeller vanes and guide vanes are taken as logarithmic spirals with no thickness.
- (5) Interaction of impeller to guide vanes is neglected, so that guide vane circulation is steady and constant.
- (6) Each impeller vane wake is replaced by a distributed free vortex which is shed from the trailing edge and carried downstream along the streamlines of steady flow with the local steady flow velocity.

Assumption (5) deserves some comment. In actuality, impeller and guide vanes interact with each other, and the distribution of vortices on the impeller and guide vanes becomes unsteady. But unless the impeller and guide vanes clearance is very small, the contribution of this interaction to the lateral fluid force is considered to be small. In this study, this interaction is neglected and the vortex distribution on the guide vanes is assumed to be steady and constant.

Figure 3 shows the centrifugal impeller considered here. The impeller is surrounded by a vane diffuser and rotates at a constant angular speed  $\omega$ , while its center  $O$  whirls at a constant angular speed  $\Omega$  along a circular orbit with a fixed center  $O'$ . The vaneless diffuser problem can be treated as a special case of the vane diffuser problem. A relative coordinate system fixed to the impeller is used in the impeller flow analysis. The absolute center  $O'$  and guide vanes rotate around the impeller center  $O$ . All vectorial quantities such as position, velocity, and force are expressed by corresponding complex numbers.

### Unsteady Flowfield

Let the position vector of  $OO'$  in the relative coordinate system be  $\epsilon$ ; then  $\epsilon$  is given by

$$\epsilon = |\epsilon| e^{i(\Omega - \omega)t} \quad (1)$$

An arbitrary point on the guide vanes rotating around  $O'$  at speed  $\omega$ , is described by:

$$z_{G0} = \epsilon + z_{G0} e^{-i\omega t} \quad (2)$$

where  $z_{G0}$  is the position vector of the same point in the absolute coordinate system.

The singularity method is applied to this problem. The flow rate  $Q$  is replaced by source  $Q/b$  at  $O'$  and impeller and guide

vanes are replaced by the distribution of vortices,  $\gamma$  and  $\gamma_G$ . When the impeller whirls, free vortices are shed from the trailing edges of the impeller vanes.

The distribution of vortices on the impeller has to satisfy the boundary condition that there is no flow through the vanes. This leads to

$$\text{Im}[\{v_Q(z_1) + v_\omega(z_1) + v_\Omega(z_1) + v_\gamma(z_1) + v_a(z_1) + v_G(z_1)\} (\bar{z}_1 / |z_1|) e^{i\beta}] = 0 \quad (3)$$

where,  $z_1$  is an arbitrary point on the vanes and  $v_Q(z_1)$ ,  $v_\omega(z_1)$  and  $v_\Omega(z_1)$  represent the complex velocities caused by  $Q$ ,  $\omega$  and  $\Omega$ , respectively.

These velocities are expressed further as

$$v_Q(z_1) = \frac{Q/b}{2\pi(\bar{z}_1 - \bar{\epsilon})} \quad (3a)$$

$$v_\omega(z_1) = -iz\omega \quad (3b)$$

$$v_\Omega(z_1) = i\epsilon\Omega \quad (3c)$$

$v_\gamma(z_1)$ ,  $v_a(z_1)$ , and  $v_G(z_1)$  represent the complex velocities induced by the distribution of vortices  $\gamma$ ,  $\gamma_a$ , and  $\gamma_G$ . They are expressed as

$$v_\gamma(z_1) = \sum \int_S \frac{-i\gamma(s)}{2\pi} \frac{1}{\bar{z}_1 - \bar{z}(s)} ds \quad (3d)$$

$$v_a(z_1) = \sum \int_{S_a} \frac{-i\gamma_a(s)}{2\pi} \frac{1}{\bar{z}_1 - \bar{z}(s)} ds \quad (3e)$$

$$v_G(z_1) = \sum \int_{S_G} \frac{-i\gamma_G(s)}{2\pi} \frac{1}{\bar{z}_1 - \bar{z}(s)} ds \quad (3f)$$

where the integral path  $S$ ,  $S_a$ , and  $S_G$  represent the contour of impeller vanes, shed free vortices and guide vanes, respectively. The unknowns in equation (3) are  $\gamma$ ,  $\gamma_a$ ,  $\gamma_G$  and the contour of free vortices  $S_a$ .

According to assumption (5), the distribution of vortices on guide vanes  $\gamma_G$  is determined from the flowfield, where the impeller vortices are replaced by a time-averaged circulation concentrated at the fixed center  $O$ .

The contour of free vortices  $S_a$  can be obtained by integrating their velocities. The strength of shed vortices from a trailing edge of impeller vane  $\gamma_a(z_T)$  can be determined from  $\gamma$  and  $\gamma_a$  by applying Helmholtz's law on the conservation of vorticity giving:

$$\frac{d\Gamma}{dt} = -\gamma_a(z_T) |dz_a/dt|_{z_a=z_T} \quad (4)$$

The above relation shows the amount of shed vortices per unit time is equal to the rate of change of impeller vane circulation.

Then the equation (3) can be considered as an integral equation to determine  $\gamma$ , since  $\gamma_a$ ,  $\gamma_G$ , and  $S_a$  are known.

### Unsteady Fluid Forces

In two-dimensional potential flow theory, the fluid force  $\mathbf{F}(F_x, F_y)$  acting on an arbitrary body in an unsteady flow is obtained by applying the conservation law of momentum to a closed loop  $C$  enclosing the body,

$$F_x - iF_y = \frac{1}{2} i\rho \oint_C (dW/dz)^2 dz - i\rho \oint_C \bar{z} d\bar{W} \quad (5)$$

where  $W$  is the complex potential of the flow field.

Applying the above equation to the whirling impeller with the integration path shown in Fig. 4, the following relation is obtained by partial integration and the Cauchy-theorem (Imaichi 1980);

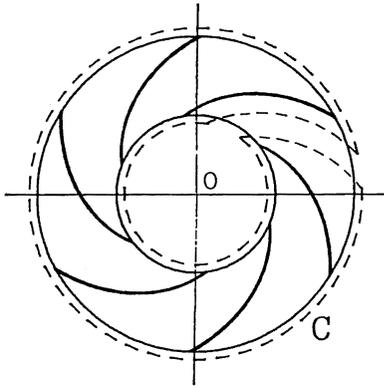


Fig. 4 Integration path

$$\begin{aligned}
 F_x - iF_y = & -i\rho \left[ \sum_S^N \gamma(l) \left\{ \frac{Q}{2\pi} \frac{1}{z(l) - \epsilon} \right. \right. \\
 & \left. \left. + \frac{i}{2\pi} \sum_{S_a}^N \frac{\gamma_a(s)}{z(l) - z(s)} ds \right\} dl \right] \\
 & - i\rho \sum_S^N \int_S \gamma(l) (-i\omega z(l) + i\Omega \epsilon) dl \\
 & + i\rho \sum_S^N \int_S \overline{(z(l) - \epsilon)} \frac{\partial \gamma(l)}{\partial t} dl \\
 & - i\rho \sum_S^N \overline{(z_t - \epsilon)} \frac{\partial}{\partial t} \int_S \gamma(l) dl
 \end{aligned} \quad (6)$$

The first term represents the forces acting on the impeller vortices from the velocity induced by  $Q$  and  $\gamma_a$ , the second term is due to the rotation and whirling motion of the impeller while the third and fourth terms represent the forces due to the change of vortex distributions.

The radial and tangential components of the fluid force,  $F_r$  and  $F_\theta$ , can be obtained from  $F_x$  and  $F_y$  by

$$F_r - iF_\theta = (F_x + iF_y) | \epsilon | / (-\epsilon) \quad (7)$$

The authors (1980) calculated unsteady force by the integration of pressure distribution on the vane surfaces. This method could be applied, only when the shockfree entry condition is satisfied. This restriction is removed by equation (6) which enable the influences of flow rate on fluid forces to be evaluated.

### Method of Numerical Calculation

The vortex distributions on vanes and wakes are replaced in the calculations by a finite number of discrete vortices. In this way the integral equations are transferred into simultaneous equations. If  $\gamma$  is known at the time  $t = t$ , the strength of vortices shed from the impeller vane trailing edges are determined from equation (4). These values are used in the above mentioned simultaneous equations to obtain  $\gamma$  at the time  $t = t + \Delta t$ , where  $\Delta t$  is an appropriate time interval. Thus, starting from the time  $t = 0$  when no free vortex exists,  $\gamma$  can be obtained stepwisely. The integrations are carried out numerically using the trapezoidal method and analytically for the free vortices in the vicinity of the trailing edges.

### Results of Whirling Impeller in Vaneless Diffuser

The case of a whirling impellers in vaneless diffuser is first calculated, by neglecting the appropriate terms of equations (3) and (6), representing the guide vane influence. The

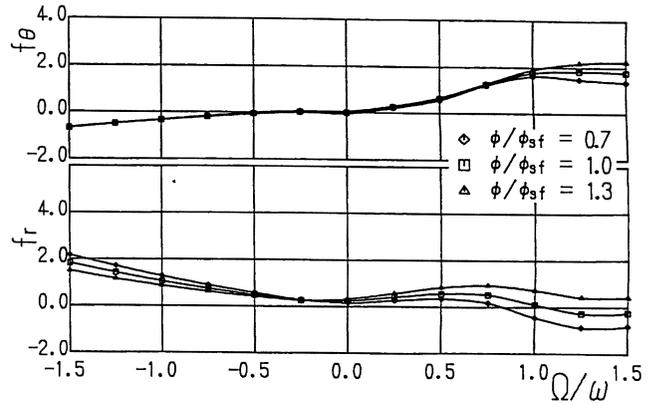


Fig. 5(a) Normalized fluid force of impeller in vaneless diffuser ( $N = 6$ ,  $\beta = 45$  deg)

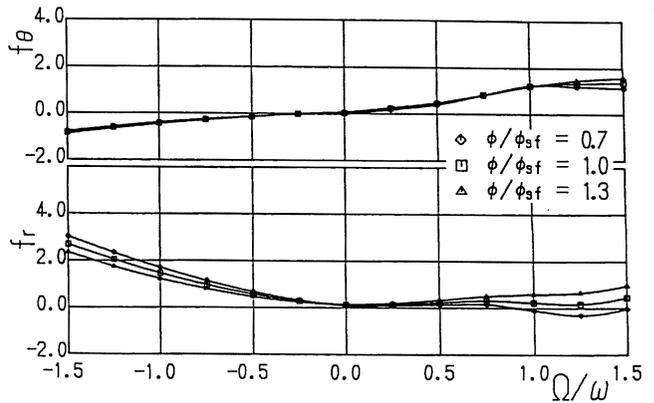


Fig. 5(b) ( $N = 6$ ,  $\beta = 60$  deg)

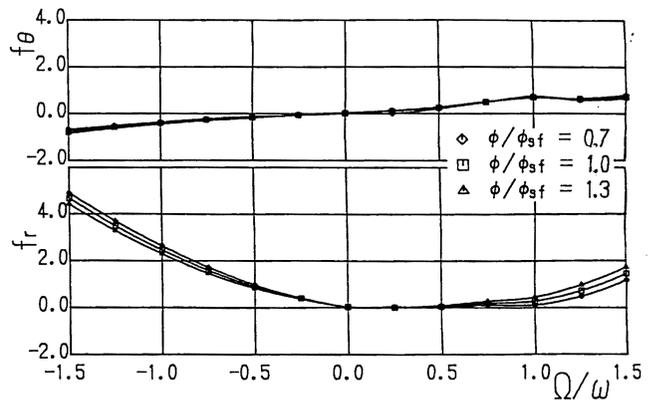


Fig. 5(c) ( $N = 6$ ,  $\beta = 75$  deg)

Table 1 Geometry of impellers for sample calculations

vane angle to radius, $\beta$ (logarithmic spiral)	75°	60°	45°
$r_1 / r_2$	0.5	0.5	0.5
number of vanes, $N$	6	3, 6, 9	6
shockfree flow coefficient, $\phi_{sf}$	0.067	0.144	0.25

calculations were performed for the conditions listed in Table 1 to understand influences of the impeller geometry and flow rate on the lateral forces.

When the number of vanes exceeds two, the magnitude of

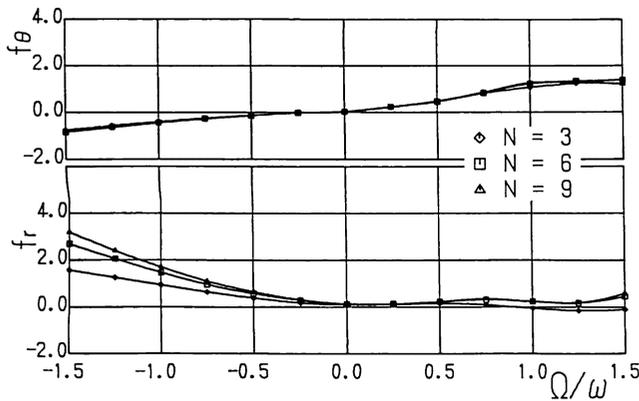


Fig. 6 Normalized fluid force of impeller in vaneless diffuser ( $\beta = 60$  deg,  $\phi/\phi_{sf} = 1.0$ )

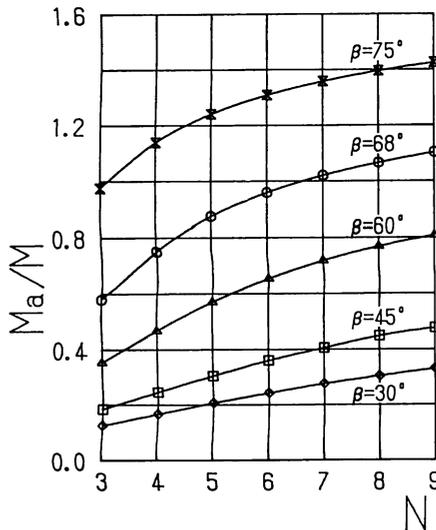


Fig. 7 Apparent mass of two-dimensional centrifugal impeller (logarithmic spiral vanes,  $r_1/r_2 = 0.5$ )

resultant force stays constant and its direction rotates with angular speed  $\Omega - \omega$  in the relative coordinate system. The force acting on individual vanes varies independently with time. The direction of resultant force in the absolute coordinate system rotates counterclockwise at angular speed  $\Omega$ , so the angle between the force vector and eccentricity  $O'O$  remains constant.

The radial force component  $F_r$  and tangential force component  $F_\theta$ , are further normalized as follows:

$$\begin{aligned} f_r &= F_r / (M|\epsilon|\omega^2) \\ f_\theta &= F_\theta / (M|\epsilon|\omega^2) \\ M &= \rho\pi r_2^2 b \end{aligned} \quad (8)$$

where  $\rho$  is the density of the fluid and  $M$  is the mass of the fluid displaced by the projected volume of impeller vanes.

Figure 5 shows  $f_r$  and  $f_\theta$  calculated for whirl speed ratio  $\Omega/\omega$  values between  $-1.5$  to  $1.5$ , when the impellers ( $\beta = 45$  deg,  $\beta = 60$  deg,  $\beta = 75$  deg) operate at three different flow rates ( $\phi/\phi_{sf} = 0.7, 1.0, 1.3$ ).

These results show that the radial force  $f_r$  increases as the flow rate increases in positive whirl, with a decrease in negative whirl. The calculated tangential force  $f_\theta$  is positive in positive whirl and negative in negative whirl. This indicates that fluid forces exert a damping effect on the whirling impeller. The influence of flow rate on  $f_\theta$  is generally small. The influence of number of impeller vanes,  $N$ , can be seen in Fig. 6, where as  $N$  increases the radial force  $f_r$  becomes larger but

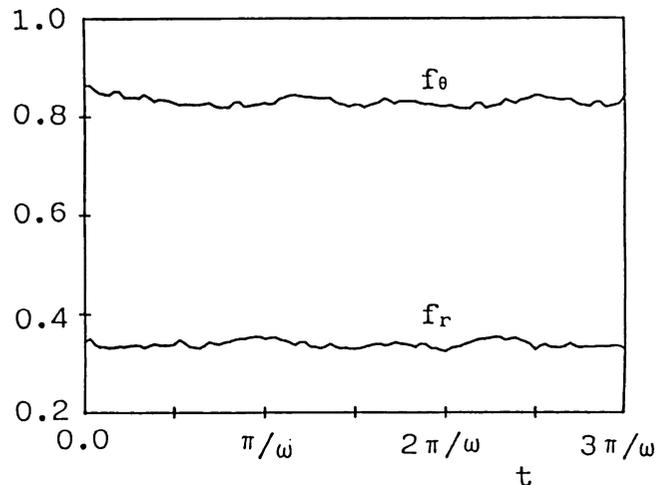


Fig. 8(a) Instantaneous values of fluid forces on impeller in vaned diffuser ( $\Omega/\omega = 0.75$ ,  $N_G = 2$ ,  $\beta_G = -75$  deg,  $|\epsilon|/r_2 = 0.001$ ,  $(r_3 - r_2)/r_2 = 0.114$ )

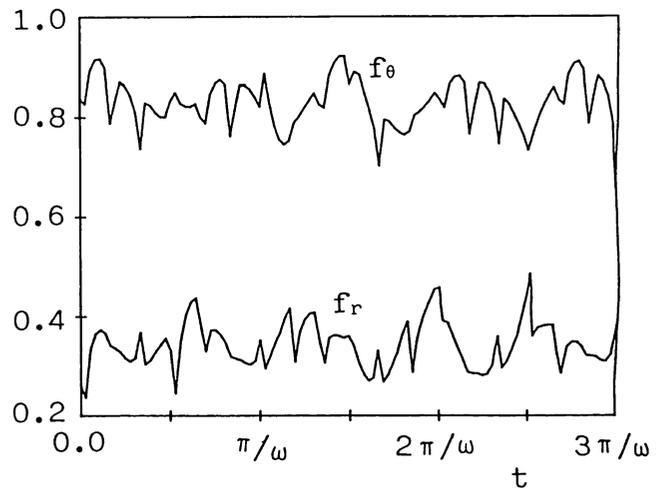


Fig. 8(b) ( $\Omega/\omega = 0.75$ ,  $N_G = 2$ ,  $\beta_G = -60$  deg,  $|\epsilon|/r_2 = 0.001$ ,  $(r_3 - r_2)/r_2 = 0.114$ )

as  $N$  increases the calculated tangential forces  $f_\theta$  appears almost independent of  $N$ . The  $f_r$  component is substantially influenced both by  $\beta$  and  $N$  since  $f_r$  is caused by the centrifugal force on the apparent mass of the impeller. Figure 7 shows the impeller apparent mass  $M_a$  calculated by potential theory for translatory oscillation without rotation. The results of Figs. 5 and 6 are consistent with these apparent mass values.

### Results of Whirling Impeller in Vaned Diffuser

The fluid forces acting on a whirling impeller surrounded by guide vanes were next calculated. In these calculations the impeller and guide vanes interaction was neglected and the circulation around each guide vane was assumed to be constant for the sake of simplicity.

When the number of guide vanes is equal to two or more, their influences on the impeller without whirling motion cancel each other and cause no lateral force. Once the whirling motion occurs, this cancellation stops and the fluid force varies in a very complicated way.

Figure 8 shows the time variation of  $f_r$  and  $f_\theta$  during whirling motion. In the case of  $\beta_G = -75$  deg (low guide vane loading),  $f_r$  and  $f_\theta$  change little as shown in Fig. 8(a). In the case of  $\beta_G = 60$  deg (high loading), a significant interaction occurs as seen from Fig. 8(b). In spite of the fluctuation of the

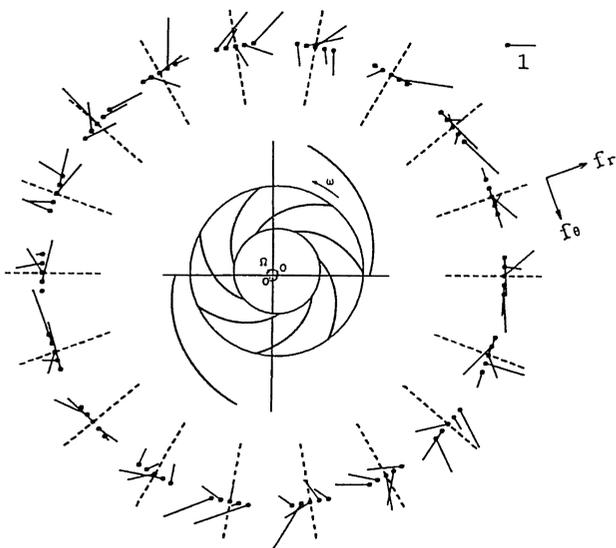


Fig. 9(a) Instantaneous fluid force vectors of impeller in vaneless diffuser at 18 equally spaced locations on the orbit ( $r_1/r_2 = 0.5$ ,  $\beta = 60$  deg,  $N = 7$ ,  $\Omega/\omega = 0.57$ ,  $r_3/r_2 = 1.05$ ,  $r_4/r_2 = 1.714$ ,  $\beta_G = -60$  deg,  $N_G = 2$ ,  $l_e/r_2 = 0.025$ ,  $N\omega/N_G\Omega = 6.14$ )

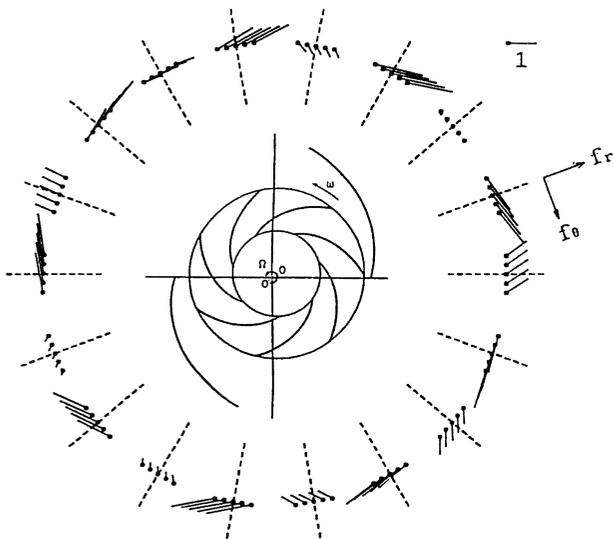


Fig. 9(b)  $\Omega/\omega = 0.7$ ,  $N\omega/N_G\Omega = 5$  (integer)

instantaneous values of  $f_r$  and  $f_\theta$ , the calculations indicate that their time-averaged values are almost unaffected.

In Fig. 9 instantaneous force vectors in the absolute coordinate system at 18 equally spaced orbit locations are shown. Five successive vectors are plotted at each location from left to right. The force vectors change randomly as shown in Fig. 9(a), because the relative position of impeller vanes and guide vanes is different every time the impeller center  $O$  passes the same location of the orbit. Only in special cases when the value  $N\omega/N_G\Omega$  is equal to an integer do the same relative position of impeller and guide vanes occur at an orbital location. In this case the successive force vectors are the same at each location as shown in Fig 9(b).

## Conclusion

This paper has presented the results of flow calculations for a two-dimensional centrifugal impeller whirling in a vaneless and a vaned diffuser. These calculations illustrate the influences of the impeller geometry, whirling speed to impeller speed ratio, flow rate, and guide vane geometry, etc. on the unsteady lateral force. The calculated results showed the following:

### (I) Whirling impeller in vaneless diffuser:

(1) The magnitude of fluid force stays constant, provided the number of vanes exceeds two, and the angle between force vector and eccentricity  $O'O$  remains constant, which is a direct consequence of the assumed circular orbit.

(2) Tangential force  $f_\theta$  is positive in positive whirl and negative in negative whirl. Thus the fluid force exerts a damping effect on whirling impeller.

(3) Radial force  $f_r$  increases in positive whirl and decreases in negative whirl as the flow rate increases. The flow rate influence on tangential force  $f_\theta$  is generally small.

### (II) Whirling impeller in vaneless diffuser:

(4) The components  $f_r$  and  $f_\theta$  undergo significant fluctuations due to impeller-guidevane interactions while the time-averaged values appear to be unaffected.

(5) The instantaneous force vectors change randomly in general whirling conditions. Only in the special cases that the value  $N\omega/N_G\Omega$  is equal to an integer do the successive force vectors at an orbital location become the same.

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## References

- Allaire, P. E., et al., 1984, "Hydraulic Forces on a Centrifugal Impeller Undergoing Synchronous Whirl," NASA CP 2338, pp. 123-135.
- Black, H. F., 1969, "Effects of Hydraulic Forces in Annular Pressure Seals on the Vibrations of Centrifugal Pump Rotors," *J. Mech. Eng. Sci.*, Vol. 11, No. 2, pp. 206-213.
- Chamieh, D., 1983, "Forces on a Whirling Centrifugal Pump Impeller," Ph.D. thesis, California Institute of Technology.
- Childs, D., and Kim, C., 1984, "Analysis and Testing for Rotordynamic Coefficients of Turbulent Annular Seals with Different, Directionally Homogeneous Surface-Roughness Treatment for Rotor and Stator Element," NASA CP 2338, 1984, pp. 313-340.
- Colding-Jorgensen, J., 1980, "Effect of Fluid Forces on Rotor Stability of Centrifugal Compressors and Pumps," NASA CP 2133, 1980, pp. 249-265.
- Domm, V., and Hergt, P., 1970, "Radial Forces on Impellers of Volute Casing Pumps," *Flow Research on Blading*, Elsevier Pub. Co., pp. 305-321.
- Hergt, P., and Krieger, P., 1969-1976, "Radial Forces in Centrifugal Pumps with Guide Vanes," *Proc. Inst., Mech. Eng.*, Vol. 184, Part 3N, pp. 101-107.
- Iino, T., and Kaneko, H., 1980, "Hydraulic Forces Caused by Annular Pressure Seals in Centrifugal Pumps," NASA CP 2133, pp. 213-225.
- Imaichi, K., et al., 1980, "A Two-Dimensional Analysis of the Interaction of Radial Impeller in Vaneless Casing," *Proc. 10th IAHR Symposium*, 1980, pp. 635-647.
- Shoji, H., and Ohashi, H., "Fluid Forces on Rotating Centrifugal Impeller with Whirling Motion," NASA CP 2133, 1980, pp. 317-328.
- Someya, T., 1987, "Journal Bearing Databook," Springer, 1987.
- Thompson, W. E., "Fluid Dynamic Excitation of Centrifugal Compressor Rotor Vibrations," *ASME JOURNAL FLUIDS ENGINEERING*, Vol. 100, No. 1, 1978, pp. 73-78.
- Tsujiyama, Y., Acosta, A. J., and Brennen, C. E., 1986, "Theoretical Study of Fluid Forces on a Centrifugal Impeller Rotating and Whirling in a Volute," *Trans. JSME*, Vol. 52, No. 480, 1986, pp. 3159-3165 (in Japanese).