

Yuji Kubota, Toshiba Corp., 1 Komukai-Toshibacho, Saiwai-ku, Kawasaki 210, JAPAN

Hideo Ohashi, University of Tokyo

#### ABSTRACT

The natural frequencies of hydraulic pumps have been studied by considering a simple theoretical model. In this paper, emphasis was placed on the effect of the relative rotation between the fluid and the pump structure. An approximate expression was derived for the natural frequencies of a structure in contact with a rotating fluid. In the theoretical model, the vibration of a disk in a fluid-filled cylindrical container was investigated. The disk was considered as a uniform annular plate with a small ring width compared with its radius. The fluid was considered to be inviscid and rotating uniformly. It was assumed that the differential coefficients of the motion of the fluid and the disk in the radial direction were negligibly small. Only diametrical modes have been discussed. The approximate expression shows that there are two natural frequencies for each diametrical mode of the pump structure. The natural frequency of the wave rotating in the same direction as the fluid rotation is higher than that of the wave rotating oppositely for each diametrical mode.

#### INTRODUCTION

The natural frequencies of hydraulic machinery such as turbo pumps are discussed in this study. Especially, the study has been focused on the diametrical modes of a hydraulic machine structure with a flat runner. Vibration problems from the diametrical modes have never occurred in conventional hydraulic machinery. However, for example, resonance problems from the diametrical modes have occurred in Francis pump-turbines recently, because the shapes of pump-turbines have become flatter as their pump heads have become higher[1]. The resonance of a hydraulic machine arises when the exciting frequency, which comes from the interaction between the runner and the stationary structure such as the guide vane and the throat of the casing, and the natural frequency of the structure coincide with each other. It has been reported that only the diametrical modes, which are determined from the combination between the number of blades of the runner and the number of the exciting sources (in this paper, stationary vanes,

throats of casings, etc. are collectively designated "exciting sources"), are excited[2]. Consequently, it is important to know the natural frequencies and their vibration modes of the structure under actual operating conditions (i.e., in contact with water) in the design stage of hydraulic machinery. Recently, the finite element method for solving fluid-structure interaction problems has been developed[3]. It enables predicting the natural frequencies of a structure in contact with a fluid accurately, if the effect of the fluid viscosity is small. In the early stages of design, however, a simplified model is more intuitive and more useful. A simple model for estimating the natural frequencies of a disk-like structure in a fluid-filled container has been presented in reference [4]. In reference [4], it has been assumed that the differential coefficients of the motion of the fluid and the disk in the radial direction were negligibly small, and that an approximate expression was derived for the natural frequencies of an annular plate in a fluid-filled cylindrical container. The usefulness of the expression was confirmed by experiment, briefly reviewed in Section 3 of this paper.

In this paper, emphasis has been placed on the effect of the relative rotation between the fluid and the pump structure on the natural frequencies of turbo pumps, and an approximate expression has been derived for the natural frequencies of a structure in contact with a rotating fluid.

#### DERIVATION OF APPROXIMATE EXPRESSION

##### Theoretical Model

The theoretical model is shown in Fig.1. In the theoretical model, the vibration of a disk in a fluid-filled cylindrical container has been investigated. The wall of the container was considered to be rigid and the fluid was regarded as inviscid. The disk was considered as a uniform annular plate with a small ring width compared with its radius. It has been assumed that the differential coefficients of the motion of the fluid and the disk in the radial direction were negligibly small and that there was no fluid flow in the gap between the disk and the container. These assumptions have been used in reference [4]. In this paper, additionally, it has been assumed

that the fluid was rotating uniformly at rotation speed  $\Omega_F$ , in order to investigate the effect of the relative rotation between the fluid and the pump structure. In Fig.1,  $r$  is the radial component,  $z$  the vertical component,  $\theta$  the circumferential component fixed on the stationary side (i.e., on the disk),  $\psi$  the circumferential component fixed on the rotating side (i.e., on the fluid) of the cylindrical co-ordinates, respectively.

#### Fluid Force Exerting to Disk

Under these assumptions, the Laplace equation for the velocity potential  $\phi$  of the fluid can be written as [4]

$$\frac{1}{r_0^2} \frac{\partial^2 \phi}{\partial \psi^2} + \frac{\partial^2 \phi}{\partial z^2} = 0 \quad (1)$$

where  $r_0$  is the averaged radius (for example,  $r_0 = (r_1 r_2)^{1/2}$ ). The velocity potential  $\phi$  must satisfy the following boundary conditions.

$$\frac{\partial \phi}{\partial z} = 0 \quad \text{at } z=0 \quad (2)$$

$$\frac{\partial \phi}{\partial z} = \frac{\partial w}{\partial t} \quad \text{at } z=H \quad (3)$$

where  $w$  is the displacement of the disk in the  $z$  direction, and  $t$  is time.

Now, assume that the disk oscillates as

$$w = \sum_{n=\pm 1}^{\pm \infty} A_n e^{jn\theta} e^{j\omega t} \quad (4)$$

where  $j = (-1)^{1/2}$ . The absolute value of  $n$  indicates the number of diametrical nodal lines and the sign of  $n$  shows the rotating direction of a wave excited on the disk. When  $n$  is negative, the wave is a forward-moving wave (positive direction of  $\theta$ ), while it is a backward-moving wave (negative direction of  $\theta$ ) when  $n$  is positive.  $A_n$  is the amplitude of oscillation. Since the relationship between  $\theta$  and  $\psi$  is

$$\theta = \psi + \Omega_F t \quad (5)$$

Eq.(4) can be rewritten by using the co-ordinates fixed on the rotating side, as follows.

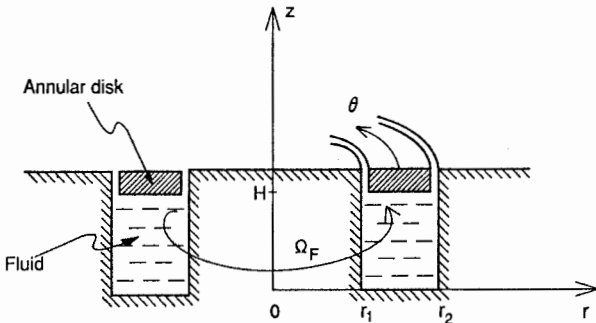


Fig.1 Theoretical Model

$$w = \sum_{n=\pm 1}^{\pm \infty} A_n e^{jn\psi} e^{j(\omega + n\Omega_F)t} \quad (6)$$

From the boundary conditions, Eqs.(2), (3) and (6), and the following orthogonal condition

$$\int_0^{2\pi} e^{j(n-m)\psi} d\psi = \begin{cases} 0 & (m \neq n) \\ 2\pi & (m = n) \end{cases} \quad (7)$$

the velocity potential  $\phi$  is obtained as follows.

$$\phi = \sum_{m=\pm 1}^{\pm \infty} \frac{j r_0 A_m}{m} (\omega + m\Omega_F) \coth\left(\frac{mH}{r_0}\right) \times e^{jm\psi} e^{j(\omega + m\Omega_F)t} \quad (8)$$

Consequently, the fluid force exerting to the fluid,  $P_{z=H}$ , can be explained as

$$P_{z=H} = -\rho_F \frac{\partial \phi}{\partial t} \Big|_{z=H} = \rho_F r_0 \sum_{m=\pm 1}^{\pm \infty} \frac{A_m}{m} (\omega + m\Omega_F)^2 \times \coth\left(\frac{mH}{r_0}\right) e^{jm\psi} e^{j(\omega + m\Omega_F)t} \quad (9)$$

where  $\rho_F$  is the density of the fluid. The translation of Eq.(9) into the stationary co-ordinates by Eq.(5) becomes

$$P_{z=H} = \rho_F r_0 \sum_{m=\pm 1}^{\pm \infty} \frac{A_m}{m} (\omega + m\Omega_F)^2 \times \coth\left(\frac{mH}{r_0}\right) e^{jm\theta} e^{j\omega t} \quad (10)$$

#### Natural Frequency of Disk

Under the assumption that the amplitude of the disk vibration is considered to be uniform in the radial direction, the equation of motion for the disk vibration can be written as [4]

$$\rho_D h_D \frac{\partial^2 w}{\partial t^2} + \frac{D^2}{r_0^4} \frac{\partial^4 w}{\partial \theta^4} = P_{z=H} \quad (11)$$

where  $D$  is the bending stiffness,  $\rho_D$  the density, and  $h_D$  the thickness of the disk, respectively.

Substituting Eqs.(4) and (10) into Eq. (11) and using Eq.(7), the following equation can be obtained.

$$[ \{ 1 + (1 + \kappa n)^2 M_0 \} \omega^2 - \omega_{n,air}^2 ] A_n = 0 \quad (12)$$

where

$$M_0 = \frac{\rho_F r_0}{n \rho_D h_D} \coth\left(\frac{nH}{r_0}\right) \quad (13)$$

$$\kappa = \frac{\Omega_F}{\omega}, \quad \omega_{n,air}^2 = \frac{n^4 D^2}{\rho_D h_D r_0^4}$$

and  $\omega_{n,air}$  is the natural frequency of the  $n$ th diametrical mode of the disk in air, and  $M_0$  is the added mass of the fluid when  $\Omega_F=0$ .

From Eq.(12), the natural frequency of the disk in contact with a fluid,  $\omega_{n,fluid}$ , can be expressed as

$$\omega_{n,fluid}^2 = \omega_{n,air}^2 / [1 + (1 + \kappa n)^2 M_0] \quad (14)$$

Equation (14) shows that the added mass of the fluid is expressed as  $(1 + \kappa n)^2 M_0$  and that the relative rotation between the fluid and the structure causes a difference in the effect of the added mass between a forward wave and a backward wave. Accordingly, there are two natural frequencies for each diametrical mode of the pump structure. The natural frequency of the forward wave ( $n$  is negative) is higher than that for the case when  $\Omega_F=0$ , while the natural frequency of the backward wave ( $n$  is positive) is lower than that when  $\Omega_F=0$ . If the phase velocity,  $-\omega/n$ , of the disk vibration and the relative rotating speed,  $\Omega_F$ , of the fluid coincide with each other (i.e.,  $\kappa n = -1$ ), the added mass effect vanishes.

In this study, though it is assumed that the fluid is rotating uniformly, distribution of the fluid rotating speed exists in an actual hydraulic machine. However, since it can be thought that the rotating speed of the fluid is uniform, except for the area near the boundary [5], it seems to be a reasonable assumption that the effect of the distribution of the fluid rotating speed on the natural frequencies is negligibly small.

#### EXPERIMENT FOR $\Omega_F=0$

In order to verify the worth and accuracy of the theoretical model in this study, an experiment carried out, as reported in reference [4], only for the case when  $\Omega_F=0$ . A brief review of the experiment follows.

#### Experimental Setup

The experimental setup is shown in Fig.2. The disk and the container were made of stainless steel. The outer and inner diameters of the disk are 405 mm and 135 mm, respectively. The disk is clamped at the inner and free at the outer. The inner diameter of the container was 407 mm, so that there was a radial clearance in width of 1 mm between the disk and the container. The rigidity

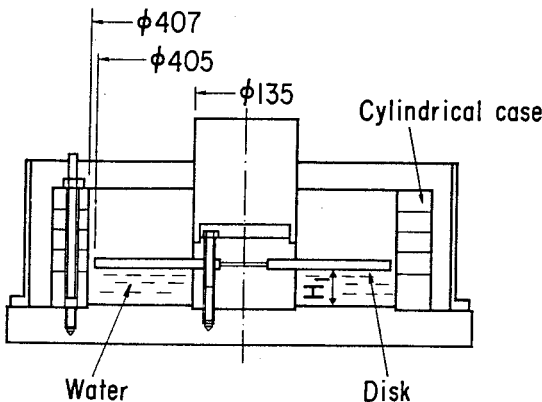


Fig.2 Experimental Setup

Table 1 Natural Frequencies of Disks in Air

| n | h = 10 | h = 5 |
|---|--------|-------|
| 1 | 346    | 194   |
| 2 | 447    | 248   |
| 3 | 792    | 412   |
| 4 | 1334   | 673   |
| 5 | 2017   | 1011  |
| 6 | 2830   | 1415  |

of the container was much higher than that of the disk, because the outer diameter of the container was 500 mm and the diameter and the thickness of the base were 620 mm and 50 mm, respectively. The fluid was water and special attention was paid to the exclusion of bubbles from the water.

The natural frequencies and vibration modes of the disk in contact with the water were measured by experimental modal analysis using an impulse hammer. Only the 1st to the 6th diametrical modes were discussed. Measurements for a 10 mm thick disk were carried out for four different water depths,  $H=10, 20, 40$  and 58 mm, and measurements for a 5 mm thick disk were carried out for two different water depths,  $H=20$  and 40 mm. The measured natural frequencies of the two disks in air are shown in Table 1.

#### Experimental Results

Figures 3 and 4 show comparisons of the results by Eq.(14) setting  $\kappa=0$  to the experimental results. In the calculation, 7.86 was used as the value of the ratio of the disk density  $\rho_D$  to the water density  $\rho_F$ .

Figure 3 shows the ratio of the natural frequencies in water to those in air,  $\omega_{n,fluid}/\omega_{n,air}$ , versus water depth  $H$ . Agreement between the results according to Eq.(14) and the experimental results is good, except for the 1st mode. In the 1st mode, the results according to Eq.(14) are smaller than the experimental results by 10 to 25 percent.

Figure 4 shows the added mass effect of water. The vertical axis is a non-dimensionalized value,  $\bar{M}_0$ , defined as

$$\bar{M}_0 \equiv \frac{\rho_D h_D}{\rho_F r_0} M_0 = \frac{1}{n} \coth\left(\frac{nH}{r_0}\right) \quad (15)$$

$\bar{M}_0$  is independent from the density and the thickness of the disk. The horizontal axis is also a non-dimensionalized value which is the ratio of the water depth  $H$  to the averaged radius  $r_0$ . In the experimental results, the value of the added mass effect ( $\bar{M}_0$ )<sub>E</sub> was defined as the following equation by using Eqs.(14) and (15).

$$(\bar{M}_0)_E = \left( \frac{\omega_{n,air}}{\omega_{n,fluid}} \right)_E^2 - 1 \quad (16)$$

where  $(\omega_{n,air}/\omega_{n,fluid})_E$  is a measured value of the ratio of the natural frequency of the  $n$ th diametrical mode in water to that in air. Both of the experimental results for  $h_D=5$  mm and  $h_D=10$  mm

are almost the same for the 3rd or more diametrical modes. Agreement between the results by Eq.(15) and the experimental results is good except for the 1st diametrical mode.

**RESONANCE MECHANISM**

In this section, a brief consideration will be given to resonance problems in a hydraulic machine corresponding to the difference between the natural frequency of a forward wave and that of a backward wave.

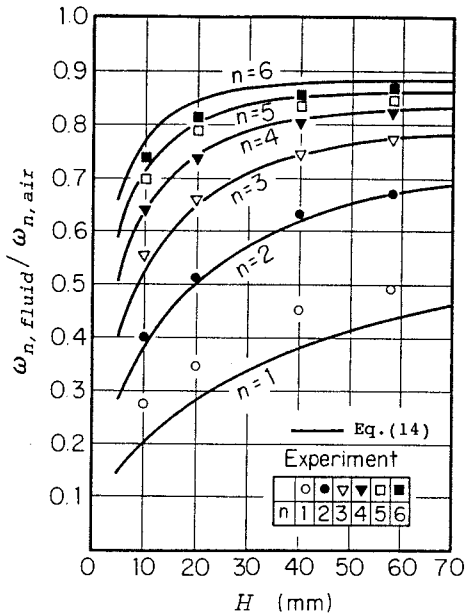


Fig.3 Natural Frequencies of Disk in Contact with Water

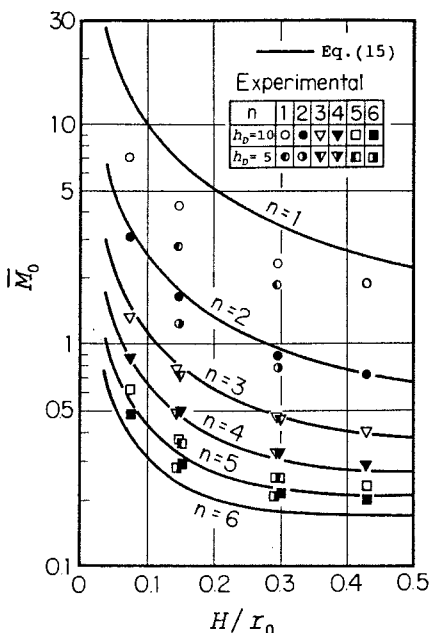


Fig.4 Added Mass Effect of Water

The frequency of a runner vibration, caused by the interaction between the runner and the stationary exciting sources,  $\omega$ , is written as

$$\omega = hZ_S \Omega_R \tag{17}$$

while the frequency of the vibration excited on the stationary structure is

$$\omega = hZ_R \Omega_R \tag{18}$$

where  $Z_R$  and  $Z_S$  are the number of blades of the runner and the stationary exciting sources respectively,  $\Omega_R$  is the rotating speed of the runner, and  $h$  is the order of harmonics of the exciting frequency ( $h=1,2,3,\dots$ ). The lower orders of  $h$  are usually important. Only the diametrical modes, which are determined from the combination between the number of blades of a runner and the number of stationary exciting sources, are excited [2]. The  $m$ th diametrical mode of a runner vibration at the frequency expressed as Eq.(17) is excited only when the following exciting condition is satisfied.

$$hZ_S \pm m = kZ_R \tag{19}$$

where  $k$  is an arbitrary integer. When  $k$  satisfies + in Eq.(19) exists, the backward wave moving in the same rotating direction (i.e., the opposite rotating direction from that of the water) of the runner is excited. On the other hand, the forward wave moving in the opposite rotating direction from that of the runner is excited when  $k$  satisfies - in Eq.(19) exists.

The  $m$ th diametrical mode of vibration of a stationary structure at the frequency expressed as Eq.(18) is excited only when the following exciting condition is satisfied.

$$hZ_R \pm m = iZ_S \tag{20}$$

where  $i$  is an arbitrary integer. When  $i$  satisfies + in Eq.(20) exists, the forward wave moving in the opposite rotating direction from that of the runner is excited on the stationary structure. On the other hand, the backward wave is excited when  $i$  satisfies - in Eq.(20). Resonances in a hydraulic machine occur only when the exciting frequency coincides with the natural frequency of the diametrical mode which satisfies either Eq.(19) or (20).

As an example, the vibration of the stationary side will be considered for the case when  $Z_R=5$  and  $Z_S=2$ . In this case,  $m=1,\dots$  for  $h=1$ , and  $m=2,\dots$  for  $h=2$ , and  $m=1,3,\dots$  for  $h=3$  satisfy both + and - in Eq.(20). Consequently, if the natural frequencies of the forward wave and the backward wave were the same, a standing wave mode resonance would be excited, when the exciting frequency coincides with the natural frequency of the  $m$ th diametrical mode. As mentioned in Section 2, however, differences between the natural frequencies of the forward wave and those of the backward wave exist because of the relative rotation between the fluid and the pump structure. Therefore, resonances of the forward wave and the backward wave occur at different frequencies

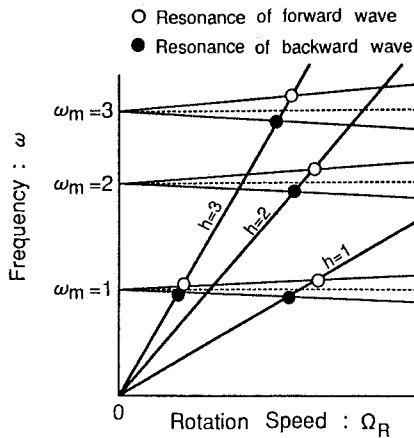


Fig.5 Resonances on Stationary Side,  
When  $Z_R=5$  and  $Z_S=2$

(i.e., the rotating speed of the runner), as shown in Fig.5. In Fig.5,  $\omega_{m=1}$ ,  $\omega_{m=2}$ , ... indicate the natural frequencies of the  $m$ th diametrical mode of a stationary structure in contact with water when  $\Omega_F=0$ .

#### CONCLUDING REMARKS

The natural frequencies of hydraulic pumps have been studied by considering a simple theoretical model. An approximate expression has been derived for the natural frequencies of a structure in contact with a rotating fluid. It has been shown that there are two natural frequencies for each diametrical mode of a pump structure, because the relative rotation between the fluid and the structure causes a difference in the effect of the added mass of the fluid between the forward waves and the backward waves. The natural frequency of the forward wave (moving in the relative rotating direction of the fluid) is higher than that of the backward wave for each diametrical mode. Therefore, resonances of the forward wave and the backward wave occur at different rotating speeds of a runner.

#### NOMENCLATURE

|                      |   |
|----------------------|---|
| $D$                  | :bending stiffness of disk  |
| $h_D$                | :disk thickness   |
| $H$                  | :fluid depth  |
| $m, n$               | :number of diametrical nodal lines  |
| $M_0$                | :added mass effect of fluid when $\Omega_F=0$                                   |
| $p$                  | :fluid pressure   |
| $r$                  | :radial component of cylindrical co-ordinates                                   |
| $r_0$                | :averaged radius  |
| $t$                  | :time   |
| $w$                  | :disk displacement  |
| $z$                  | :vertical component of cylindrical co-ordinates                                 |
| $Z_R$                | :number of runner blades  |
| $Z_S$                | :number of exciting sources   |
| $\theta$             | :circumferential component fixed on stationary side of cylindrical co-ordinates |
| $\kappa$             | : $=\Omega_F/\omega$  |
| $\rho_D$             | :disk density   |
| $\rho_F$             | :fluid density  |
| $\phi$               | :velocity potential of fluid  |
| $\psi$               | :circumferential component fixed on rotating side of cylindrical co-ordinates   |
| $\omega$             | :angular frequency  |
| $\omega_m, \omega_n$ | : $m$ th, $n$ th natural frequency  |
| $\Omega_F$           | :relative rotation speed of fluid   |
| $\Omega_R$           | :rotating speed of runner   |

#### REFERENCES

1. H. Tanaka, T. Nagafugi and I. Yamagata, Turbo Machine, Vol.13, No.10, pp.37-42, 1985 (in Japanese).
2. Y. Kubota, T. Suzuki, H. Tomita, T. Nagafugi and C. Okamura, Bulletin of the JSME, Vol.26, No.221, pp.1952-1957, 1983.
3. R.H. MacNeal, R. Citerley and M. Chargin, 5th SMiRT, B4-9, 1979.
4. Y. Kubota and T. Suzuki, Transactions of the JSME, Vol.50, No.449, C, pp.243-248, 1984 (in Japanese).
5. J.W. Daily and R.E. Nece, Trans. ASME, Ser. D, Vol.82-1, pp.217-232, 1960.